

Calculating of a spatial cantilever truss natural vibration frequency with an arbitrary number of panels: analytical solution

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Abstract:

The research object was a spatial cantilever statically determinate truss composed of three planar trusses with a triangular lattice. The spectrum of natural frequencies of the structure was analyzed. The same concentrated masses model the inertial properties of the truss at the nodes. The task was to obtain an analytical dependence of the lowest vibration frequency of a truss on the number of panels, mass, linear dimensions of the structure, and material properties. **Method.** The induction method and the Maple computer mathematics system operators were used to determine the forces in the rods and generalize the result to an arbitrary number of panels. The problem was solved using the Dunkerley approach, which gives a lower frequency estimate. Maxwell-Mohr's formula determines the rigidity of the structure. Homogeneous linear recurrent equations were compiled and solved to find the common members of the sequences of coefficients in the formula for the frequency. **Results.** The accuracy of the formula obtained by the Dunkerley method was estimated from comparison with a numerical calculation of the entire spectrum of natural frequencies. The comparison shows the good accuracy of the derived formula. As the number of panels increases, the accuracy of the lower estimate increases too. The same frequency for all trusses was found in the spectra of trusses with a different number of panels. This frequency is the spectral constant and depends only on the size of the system, the stiffness of the members, and the mass. The existence of spectral isolines with the property of asymptotically tending to a certain constant value was shown.

1 Introduction

Frequency analysis of structures is a necessary component of any dynamic analysis. In cases where only the first (fundamental) frequency is required to calculate the performance of the system, it does not make sense to calculate the entire spectrum. Numerical methods are used to calculate natural frequencies [1]–[5]. With the development of systems of computer mathematics [6], [7], it became possible to obtain an analytical expression for the first frequency, depending on the size, mass, and properties of the material of the structure. In [8]–[12] compact formulas were obtained for calculating the rigidity of some plane statically definable trusses. Based on these formulas, solutions for vibration frequencies are obtained. Frequency analysis of bridge trusses is also performed when examining steel bridges, including operated railway ones, where periodic dynamic loads are especially significant [13]. Accurate calculation of natural vibration frequencies is also very important in seismic analysis of buildings and structures [14]–[17]. In road construction, natural vibration frequencies using design formulas are required when assessing the dynamic coefficient of the road surface [16].

In analytical calculations of natural frequencies, two types of solutions can be distinguished. Most often, semi-empirical or approximate formulas are used. For example, the method of equivalent structural stiffness is used to calculate the vibration frequency of a girder [4], [18], [19]. For this, the rigidity of the truss is equated to the rigidity of the equivalent beam, for which its natural frequency is determined by

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the methods of resistance of materials. Another type of solution relates to regular structures with a periodic structure, in particular to trusses. Here, by induction, you can obtain a solution for a whole class of structures by entering as a parameter the number of periodic elements, for example, panels, or the number of rods. If the construction is statically definable, then the analytical solution has the form of a relatively simple formula [20]. Hutchinson, R.G., Fleck, N.A [21] and Zok, F.W., Latture, R.M., Begley, M.R.[22]. Analytical solutions can be used in optimization problems [15], [23]–[26], for a preliminary assessment of the properties of the designed structure.

In this paper, we consider the spectrum of natural vibrations of the spatial model of the console, the inertial properties of which are modeled by lumped masses in the hinges. The formula for the first frequency is displayed depending on the number of panels. We use Dunkerley's approximate approach, which gives a lower bound for the first vibration frequency [27]–[29].

2 Methods

2.1 The scheme of the truss. The Dunkerley's formula

The cantilever truss is fixed on a spherical support A, cylindrical B, and on a horizontal rod at the lower belt node C (Fig. 1).

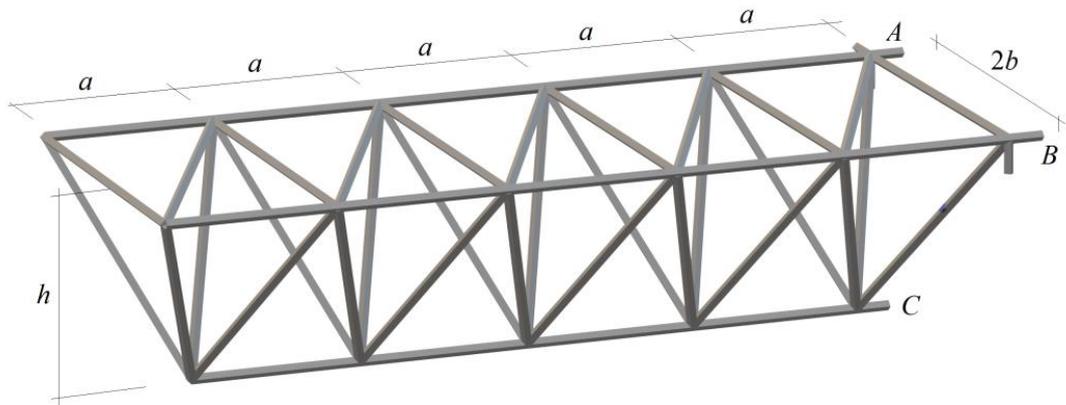


Fig. 1. Truss, $n=5$

The sides of the truss are two identical planar trusses with a triangular grid. The upper horizontal truss panel is a truss with a rectangular grid. In a truss with n panels, counting along the upper belt, $N = 9n + 6$ rods. This number also includes six support rods, the deformations of which we will neglect, considering them to be rigid. A similar spatial rod system was used in [19] to simulate antenna oscillations.

The mass of the truss is distributed among its nodes. It is assumed that the displacements of the nodes are vertical. The number of system degrees of freedom in this setting is equal to $K = 3n$.

To obtain the desired frequency dependence on the number of panels, we use the approximate Dunkerley method [20]

$$\omega_D^{-2} = \sum_{p=1}^K \omega_p^{-2}, \tag{1}$$

where ω_p is the partial frequencies. The equation of oscillations of one mass has the form

$$m\ddot{z}_p + D_p z_p = 0, \quad p=1, \dots, K, \tag{2}$$

Here z_p is the vertical coordinate of the moving mass, \ddot{z}_p is its acceleration. The stiffness D_p is the inverse of the flexibility $\delta_p = 1 / D_p$, which depends on the position of the mass on the truss, and is calculated by the Maxwell-Mohr's formula

$$\delta_p = 1 / D_p = \sum_{j=1}^{N-6} \left(S_j^{(p)} \right)^2 l_j / (EF). \tag{3}$$

The following symbols are introduced: $S_j^{(p)}$ – the force in the member with the number j from the action of the vertical unit force applied to the node where the mass is located, l_j – the length of the rod j . Each mass has its own stiffness coefficient and its own (partial) frequency. In the case of harmonic oscillations $z_p = U_p \sin(\omega t + \varphi)$ from (2) follows $\omega_p = \sqrt{D_p / m}$. Substituting this expression in (1), we get:

$$\omega_D^{-2} = m \sum_{p=1}^K \delta_p = m\Delta. \tag{4}$$

2.2 Calculation the forces in the members

To determine the rigidity of the structure, you will need to calculate the forces in the members. To do this, use the program in the language of computer mathematics Maple [6], [7]. The program includes a method for cutting out nodes. The matrix of the system of equilibrium equations of the nodes is compiled from the data on the coordinates of the hinges. Nodes (hinges) and rods are numbered (Fig. 2). The origin is selected in the support A. In this case, the coordinates have the form:

$$\begin{aligned} x_i &= a(i - 1/2), y_i = b, z_i = 0, i = 1, \dots, n, \\ x_{i+n} &= x_{i+2n+1} = a(i - 1), \\ y_{i+n} &= 0, y_{i+2n+1} = 2b, \\ z_{i+n} &= z_{i+2n+1} = h, i = 1, \dots, n + 1. \end{aligned} \tag{5}$$

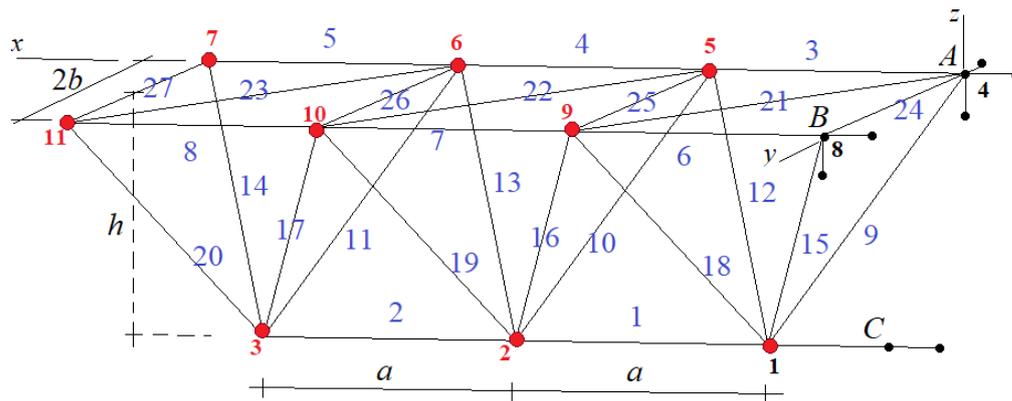


Fig. 2. Numbers of truss nodes and rods, $n = 3$

The order of the connections of the truss members is set by ordered lists $T_i, i = 1, \dots, N$, containing the node numbers at the ends of the members. The longitudinal horizontal members of the belts are encoded as follows:

$$\begin{aligned} T_i &= [i, i + 1], i = 1, \dots, n - 1, \\ T_{i+n-1} &= [i + n, i + n + 1], \\ T_{i+2n-1} &= [i + 2n + 1, i + 2n + 2], i = 1, \dots, n. \end{aligned} \tag{6}$$

The members of the grid panels are coded:

$$\begin{aligned} T_{i+jn-1} &= [i, i + n + j - 3], i = 1, \dots, n, j = 3, 4, \\ T_{i+jn-1} &= [i, i + 2n + j - 4], i = 1, \dots, n, j = 5, 6. \end{aligned} \tag{7}$$

These data are used to calculate the guiding cosines of the unknown forces that make up the matrix of the equilibrium equations of the nodes in the projections on the coordinate axis. The resulting system

of linear equations is solved in symbolic form in the Maple system. Calculations show that for an arbitrary number of panels, the solution for the coefficient Δ in (4) has the form

$$\Delta = n(C_1a^3 + C_2b^3 + C_3c^3 + C_4d^3) / (EFh^2). \tag{8}$$

The coefficients $C_i, i = 1, \dots, 5$ depend on the number of panels n . We have the following expressions

$$\begin{aligned} n = 1: \quad \Delta &= (8a^3 + 72b^3 + 9c^3 + 8d^3) / (16h^2EF), \\ n = 2: \quad \Delta &= (25a^3 + 52b^3 + 7c^3 + 6d^3) / (4h^2EF), \\ n = 3: \quad \Delta &= (164a^3 + 136b^3 + 19c^3 + 16d^3) / (16h^2EF), \\ n = 4: \quad \Delta &= (193a^3 + 84b^3 + 12c^3 + 10d^3) / (2h^2EF), \\ &\dots \end{aligned} \tag{9}$$

Using the `rgf_findrecur` and `rsolve` operators of the Maple system, you can find common terms of coefficient sequences:

$$C_1 = (3n^3 + 1) / 8, C_2 = (5 + 4n) / 2, C_3 = (4 + 5n) / 16, C_4 = (n + 1) / 4. \tag{10}$$

Taking into account (4), we get from here the desired dependence of the first frequency on the number of truss panels and its size:

$$\omega_D = 4h \sqrt{\frac{EF}{n(6a^3n^3 + (32b^3 + 5c^3 + 4d^3)n + 2a^3 + 40b^3 + 4c^3 + 4d^3)m}}. \tag{11}$$

2.3 Numerical verification

Formula (11) is an analytical approximate solution of the problem, an estimate from below. The error of the formula can be estimated by comparing it with the numerical solution of the problem of the oscillation of a system with $K = 3n$ degrees of freedom.

The solution of the natural frequencies spectrum problem of system oscillations with many degrees of freedom is reduced to the eigenvalue problem. The system of differential equations of structure vibrations with K number of degrees of freedom is written in matrix form:

$$m\mathbf{I}_K \ddot{\mathbf{Z}} + \mathbf{D}_K \mathbf{Z} = 0, \tag{12}$$

where \mathbf{Z} is the vector of all vertical mass displacements at the truss nodes, $\ddot{\mathbf{Z}}$ is the acceleration vector, \mathbf{I}_K is the unit matrix, and \mathbf{D}_K is the stiffness matrix. For harmonic oscillations with frequency, the relation $\ddot{\mathbf{Z}} = -\omega^2 \mathbf{Z}$ is valid. The matrix \mathbf{D}_K is the inverse of the flexibility matrix \mathbf{B}_K , whose elements are calculated using the Maxwell-Mohr formula:

$$b_{i,j} = \sum_{\alpha=1}^{N-6} S_{\alpha}^{(i)} S_{\alpha}^{(j)} l_{\alpha} / (EF). \tag{13}$$

If we multiply (9) by \mathbf{B}_K , we get the eigenvalue problem: $\mathbf{B}_K \mathbf{Z} = \lambda \mathbf{Z}$, where $\lambda = 1 / (\omega^2 \mu)$ are the eigenvalues of the matrix \mathbf{B}_K . For $n=1$, the matrix has the form

$$B_3 = \frac{1}{16h^2EF} \begin{bmatrix} 8b^3 + c^3 & c^3 & 16b^3 + c^3 \\ c^3 & 8a^3 + 32b^3 + 4c^3 + 4d^3 & -4d^3 \\ 16b^3 + c^3 & -4d^3 & 32b^3 + 4c^3 + 4d^3 \end{bmatrix}. \tag{14}$$

It is not possible to get eigenvalues in analytical form. The eigenvalues of the matrix are determined numerically in the Maple system by the Eigenvalues operator from the LinearAlgebra package.

3 Results and Discussion

3.1 Example of the calculation. The spectral constant. Spectral isolines

Consider a steel truss with masses $m=1200kg$ at the nodes. Take modulus of elasticity $E=2.1\cdot 10^5$ MPa, $F=50.0sm^2$, $a=3m$, $h=0.4m$, $b=1.5m$. Spectra of twelve trusses are marked in figure 3. Frequency spectra in ascending order, then the k -axis – number of frequencies. The points of the individual spectra are connected by conditional polylines $n=1, n=2, \dots, n=12$. We note some of the detected patterns of frequency distribution in the spectrum. First, it is the coincidence of frequencies for different trusses. Starting with a truss with three panels, all frequencies numbered $2n+1$ in each spectrum are equal $\omega_{2n+1}^{(k)}=203.37s^{-1}$, $k=3,4, \dots$. This frequency is called the spectral constant of the set of regular trusses. The upper index in parentheses indicates the number of panels in the truss, the lower — the number of frequencies in the spectrum. Secondly, the highest frequency of vibration is almost independent of the number of panels. Also, regardless of the number of panels in the truss, there is a sharp jump in the frequency distribution between the values $\omega_{2n}^{(k)}$ and $\omega_{2n+1}^{(k)}$. Another interesting property of the spectrum of the regular truss under consideration is almost imperceptible. We introduce the concept of a spectral isoline — a curve in the axes "number of panels – frequency", each point of which corresponds to a frequency with a number $2n+r$, $r=-2n+1, \dots, n$ in the spectrum (Fig. 4). The numerical calculation shows that when $r > 0$ the spectral isolines asymptotically tend to a horizontal line parallel to the isoline of the spectral constant. A similar picture of the asymptotic tendency can be constructed for $r < 0$.

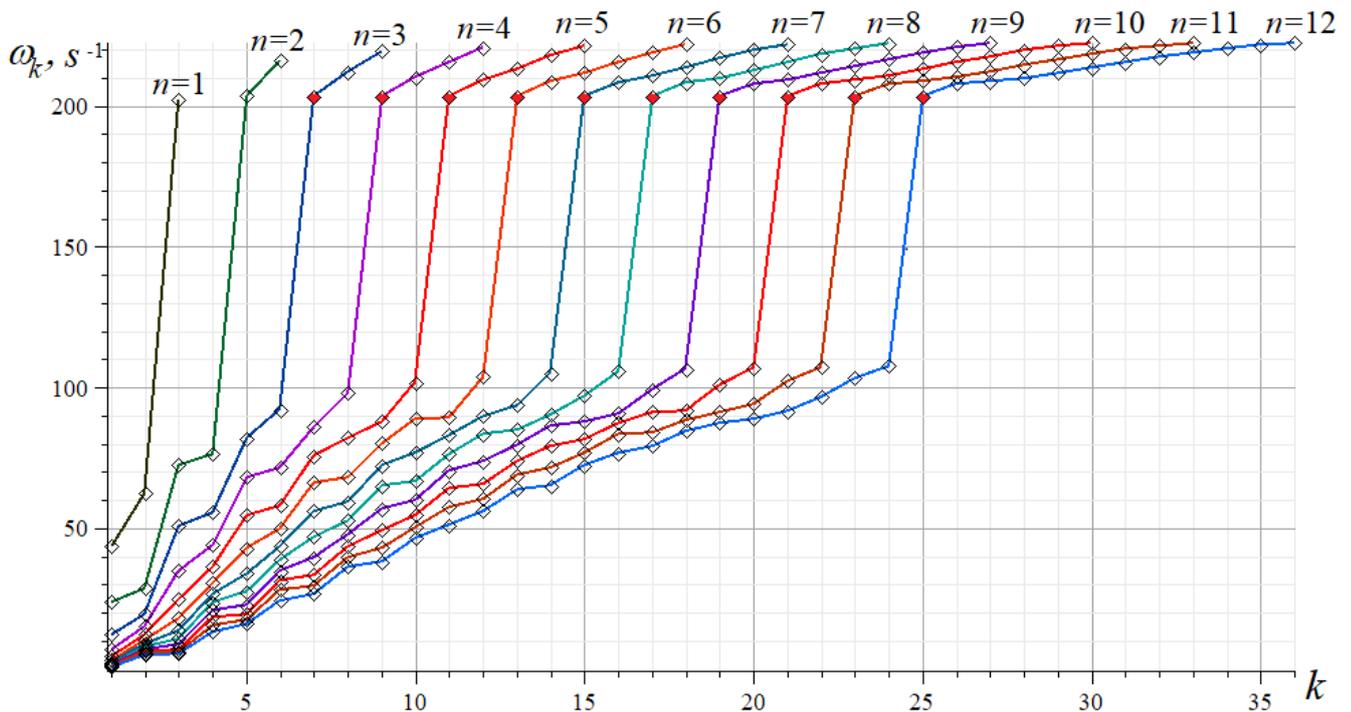


Fig. 3. Spectra of natural frequencies of trusses $n = 1, 2, \dots, 12$

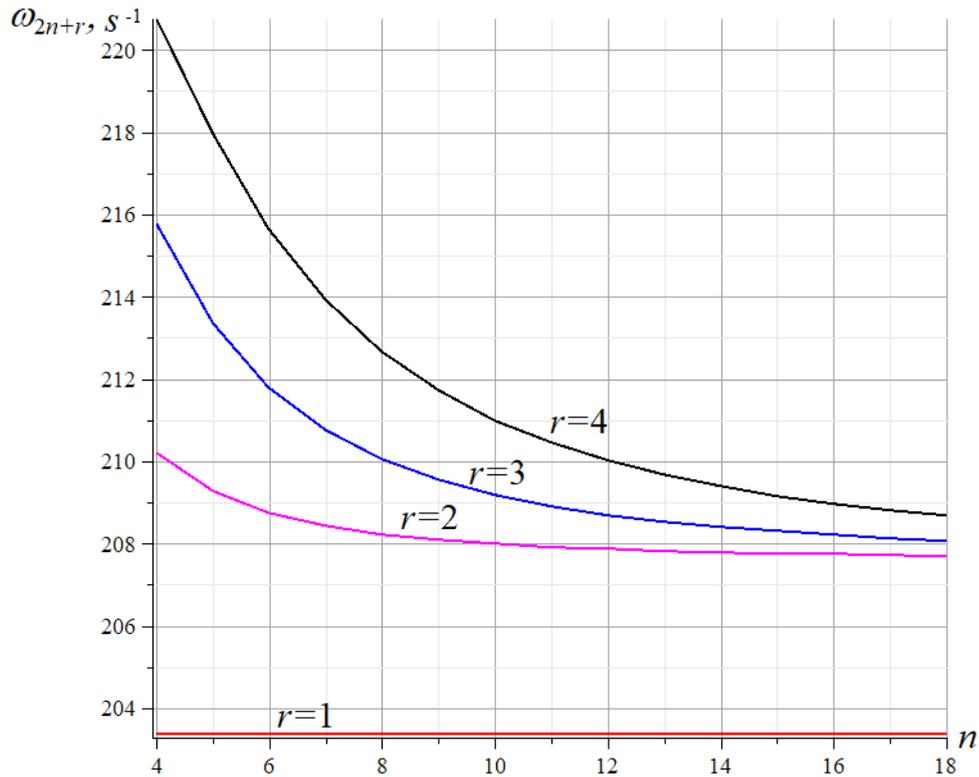


Fig. 4. Isolines of natural frequencies spectra of trusses $n = 4, \dots, 18$

Let us consider separately the value of the first frequency and compare it with the solution (11). Figure 5 shows the dependence of the frequency ω_1 , obtained numerically, and the frequency ω_D by formula (11) on the number of panels with the same data as in Figure 3. The Dunkerley curve is regularly located below the numerical solution, approaching it with an increase in the number of panels.

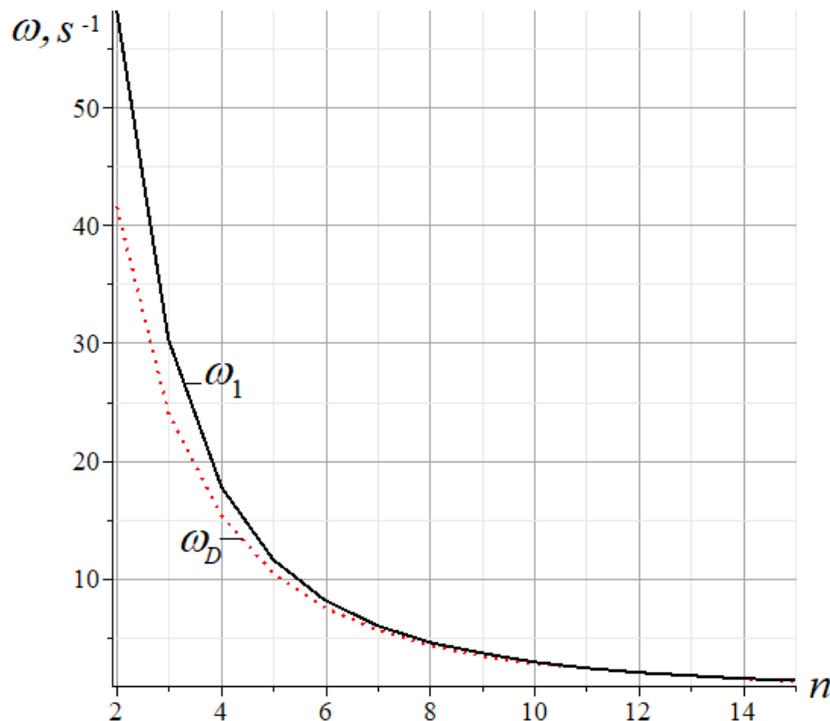


Fig. 5. Comparison with numerical calculation

For large n , the curves almost merge and it is difficult to estimate the nature of the convergence of the curves. Let us introduce a value $\varepsilon = (\omega_1 - \omega_D) / \omega_1$ to estimate the relative error of the solution.

Figure 6 shows the curves of the dependence of the error on the number of panels at three values of the height h .

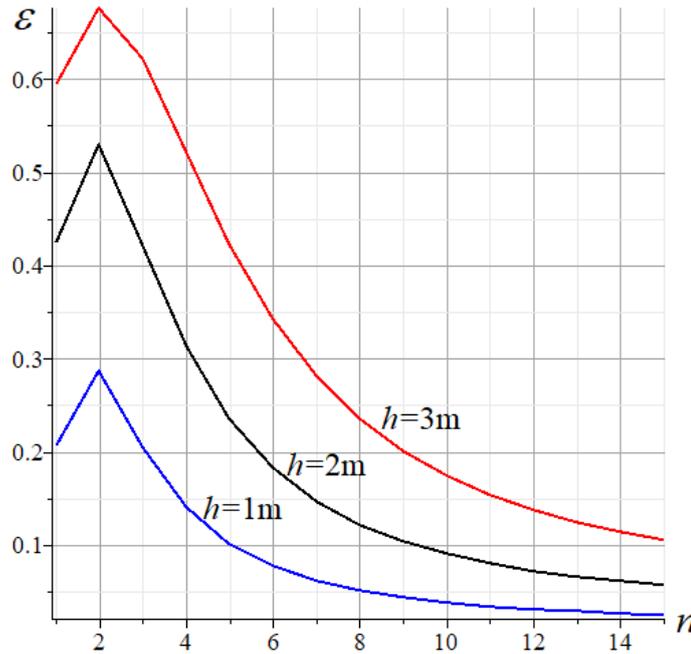


Fig. 6. Relative solution error

Except for a small spike at the beginning, at $n=1$, $n=2$, it can be seen that with an increase in n , the error ε first falls quickly, and then slowly, reaching quite satisfactorily an accuracy of several percent. Thus, the resulting analytical solution, in comparison with the numerical one, behaves in some sense in the opposite way. The numerical solution with an increase in the order of systems of equilibrium equations naturally loses accuracy due to the inevitable accumulation of rounding errors and requires more computational resources. The constructed analytical solution with an increase in the number of panels n gives greater accuracy.

Formula (11) gives a solution to the problem. A comparison of the solution with the numerical one obtained without using the approximate Dunkerley approach from the analysis of a system with many degrees of freedom revealed its high accuracy. At the same time, consideration of the entire spectrum of natural oscillations revealed several interesting properties of the frequency spectrum of the studied regular system. Some properties of the spectrum can be used in practice. So, for example, if you want to find the highest frequency of vibrations in a truss with a large number of panels, then using the fact that the value of this frequency changes slightly when n changes (increases), you can get its lower bound on the highest frequency obtained at $n=1$ from the values of the eigenvalues of the matrix (14).

Note that an alternative to the Dunkerley method in such problems is the Rayleigh [30] method, which gives an upper bound for the first oscillation frequency. The accuracy of this method is noticeably higher; however, in practice, the formulas obtained turn out to be very cumbersome and inconvenient to use.

4 Conclusions

Main results of the work are as follows.

1. The dependence of the fundamental frequency of natural oscillations of the truss on the number of panels is obtained in an analytical form by the method of induction with the involvement of a computer mathematics system based on the Dunkerley approach.
2. The accuracy of the solution increases with the number of panels, which allows us to use the resulting formula to evaluate numerical solutions for a large number of panels, i.e. in cases where the numerical solution is most time-consuming, and the possibility of counting errors is highest due to the accumulation of rounding errors.
3. In the frequency spectrum, a frequency is found that is the same for trusses with any number of panels (spectral constant).

4. Based on the numerical analysis of the frequency spectra, the existence of spectral isolines is shown.

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