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Deformation of the Transmission Towers: Analytical Solution

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Abstract:

The object of the study is a spatial model of a statically definable power line support truss. The four-sided truss has a cross-shaped lattice and a pyramidal extension at the base. In the upper part of the truss, there are consoles for attaching the carrier cables. The corner nodes in the base are attached to the ground by one spherical joint, a cylindrical joint, and two vertical posts. Two types of loads are considered: a horizontal load evenly distributed over the nodes of one face (wind), and a vertical load applied to one of the consoles. The aim is to determine the analytical dependence of the deflection of the structure on the number of mast panels in its middle part. **Method.** To determine the deflection, the Mohr integral is used. The forces in the rods are located simultaneously with the reactions of the supports from the general system of linear equilibrium equations of all nodes. Obtaining a solution and generalizing it to an arbitrary number of panels is obtained by induction in the Maple computer mathematics system. **Results**. The dependence of the deflection of the console and the displacement of the mast top on the number of panels is obtained in the form of a formula containing up to eight coefficients in the form of polynomials in the number of panels of degree no higher than the fourth. The analytical dependences of the forces in some rods as a function of the number of panels are determined. Cubic asymptotics of the solutions is found.

1 Introduction

Unlike bridges and coverings of buildings and structures, whose structures can be monolithic, beam-type, arches, and trusses, only trusses are used in power transmission poles. The calculation of transmission line trusses is usually performed numerically in various specialized engineering packages based on the finite element method [1]–[7]. However, the calculation of a statically definable or even simple statically indeterminate truss with a specific architecture is possible in analytical form. The disadvantage of such solutions, which are rigidly tied to a specific design and have only a few free independent parameters (dimensions, material constants, numerical value of the load), is that these solutions are highly specialized and have limited application. Let's consider regular constructions that have some periodic structures. The selection of options for the designed structures is possible if the analytical solution includes the order of the system, for example, the number of panels. One method of solving this problem is the induction method [8]. This method provides a number of solutions to the statics of planar arches [9]–[12], lattice trusses [13]–[19], and spatial trusses [20]. In [21], [22], the eigenfrequencies of regular hinge-rod structures are obtained. General questions about the existence and calculation of statically definable regular trusses were raised and partially solved in the articles [23], [24].

2 Materials and Methods

The structure of the tower has a cross-shaped grid and consists of two parts (Fig. 1, 2). In the lower pyramidal part with a height of *kh*, the slope of the faces is determined by the ratio of the horizontal dimensions a, c, and the vertical size *h* (Fig. 3). The upper part with parallel vertical edges consists of *n*–1 identical belts and a vertex belt with a height of h_1, on which two identical consoles with a length *r* of six rods each are fixed (Fig. 4). In total, the truss contains $m = 12(n+k)+27$ rods, including seven support ones. The corner joints at the base are attached to the ground by a spherical joint *A*, a cylindrical joint *B*, and two vertical posts *C* and *D*. Two loading options are considered. The first option (Fig. 1) is the vertical load on the console. The task is to determine in an analytical form the deflection of the console and the horizontal displacement of the vertex. The second task is to determine the horizontal displacement of the vertex under the action of a lateral horizontal load evenly distributed over the nodes, simulating a wind load.

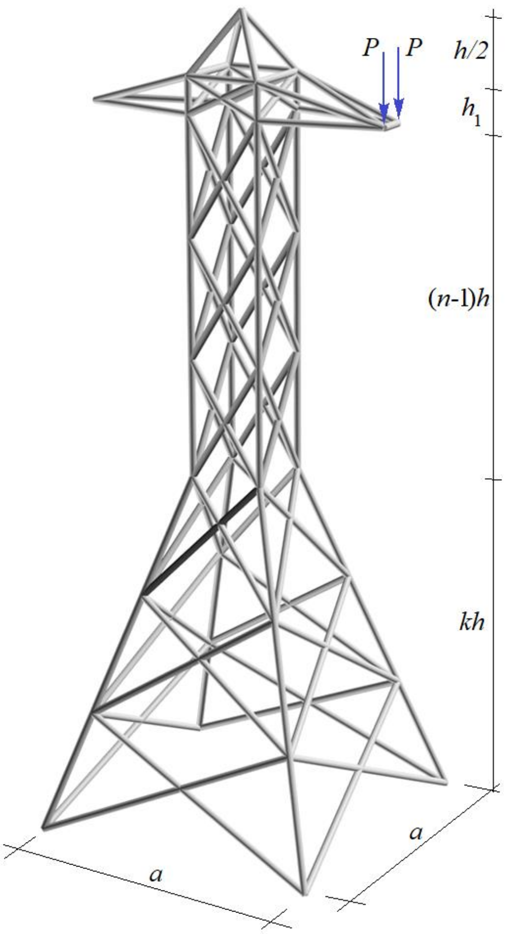

Fig. 1. Vertical load on the console, truss, *n*=4, *k*=3

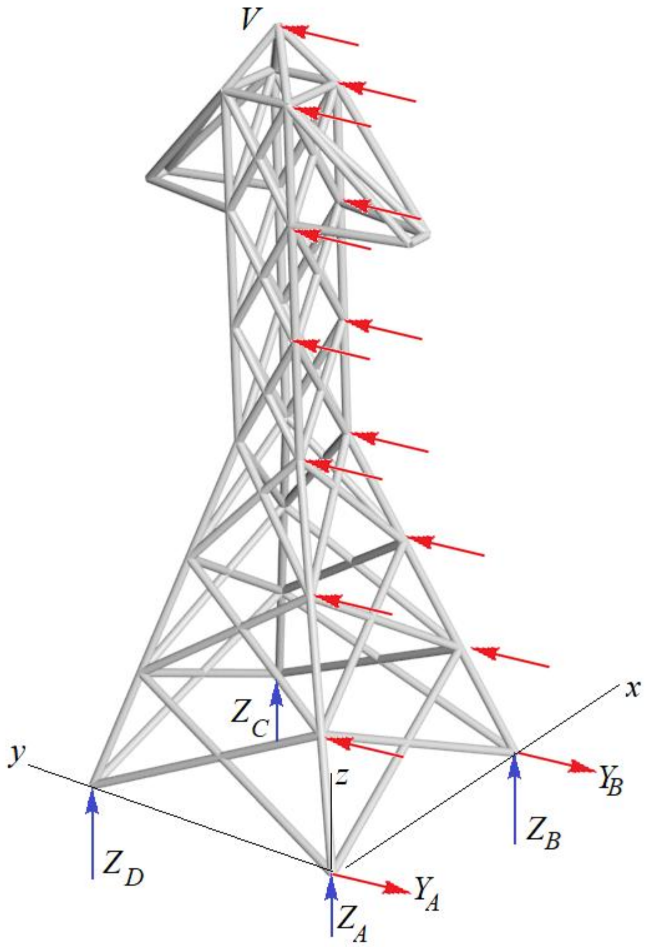

Fig. 2. Wind load, truss *n*=3, *k*=3

The forces calculation is performed in a program written in the symbolic mathematics language Maple [25], [26]. The program enters the coordinates of the nodes and the order of connection of the rods in the hinges. The graph encoding method used in discrete mathematics is used. Each rod is assigned an ordered list of the numbers of its ends. Using this data, the guiding cosines of the forces connected at the nodes are calculated. The matrix $\mathbf{G}$ of the system of equilibrium equations of nodes consists of these values. The loads on the nodes are added to the right side of the matrix equation $\mathbf{GS} = \mathbf{T}$. Here $\mathbf{S}$ is the vector of the values of all the forces in the rods, including the reactions of the supports.

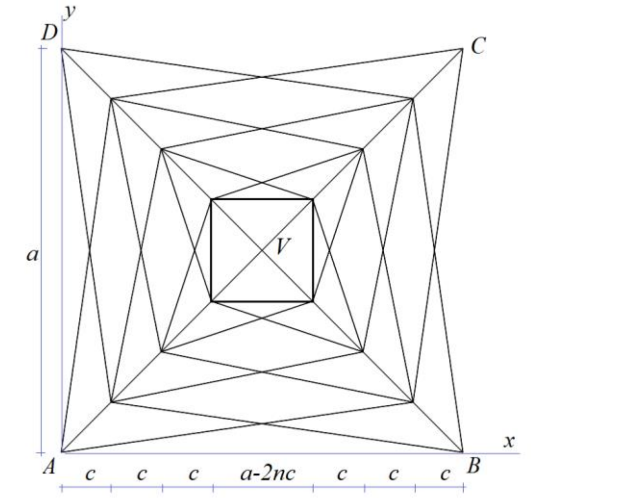

Fig. 3. Truss dimensions in plan

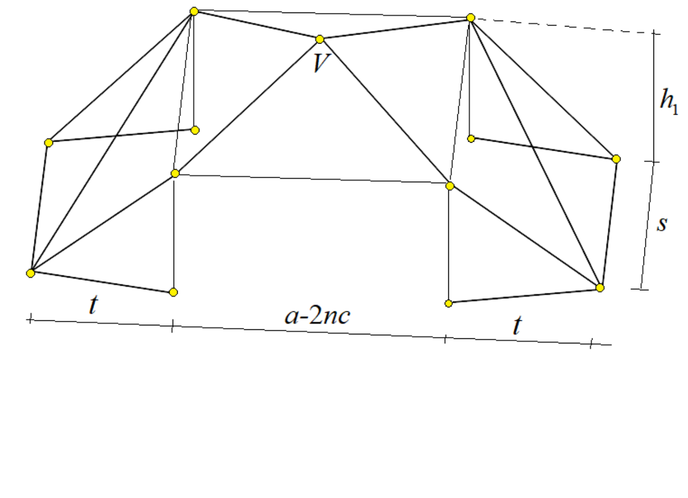

Fig. 4. Cantilever parts of the mast

Each node is assigned three rows of the matrix. The system of equations is solved in symbolic form. The movements of the truss nodes are calculated using the Maxwell-Mohr formula

$$\Delta=\sum_{\alpha=1}^{m-7} S_\alpha^P S_\alpha^1 l_\alpha /(EF). \tag{1}$$

Here it is indicated S_α^1 — the forces in the element numbered from the action of the unit force applied to the node in the direction of the desired displacement, S_α^P the force from the action of the load, *EF* —the longitudinal stiffness of the rods, l_α — the lengths of the rods. In the considered statement, the stiffness of the rods is assumed to be the same. Summation is performed for all deformable truss rods. This sum does not include the seven reference elements that are assumed to be non-deformable.

3 Results and Discussion

3.1 Calculation of deflections and displacements

3.1.1 Console deflection

As a load, consider two vertical forces *P* applied to the console. In this case, the nonzero elements of the load vector in the right part of the system of equilibrium equations of the nodes have the form

$$T_{3i}=-P,\ i=4(n+k)+8,\ \ 4(n+k)+9. \tag{2}$$

The calculation of trusses with a different number of panels *n* in the upper part of the mast at *k*=3 shows that the form of the solution for the deflection calculated by the formula (1) has the same form:

$$\Delta=P(C_1a^3+C_2r^3+C_3f^3+C_4g^3+C_5h^3+C_6p^3+C_7q^3)/(EFh^2), \tag{3}$$

The following symbols are introduced

$$r=\sqrt{50a^2+64h^2},\ f=\sqrt{10a^2+64h^2},$$
$$g=\sqrt{a^2+16h^2},\ p=\sqrt{2a^2+64h^2},$$
$$q=\sqrt{26a^2+64h^2}.$$

The coefficients $C_1,...,C_7$ in (3) depend only on *n*. These dependencies are determined by the induction method. We have the following sequence of solutions

$$\Delta_1=P(45504a^3+900f^3+70272g^3+3833856h^3+6304p^3+100q^3+25r^3)/(147456Hh^2EF)$$
$$\Delta_2=P(22464a^3+900f^3+96768g^3+9732096h^3+6304p^3+100q^3+25r^3)/(147456Hh^2EF),$$
$$\Delta_3=P(45504a^3+900f^3+169344g^3+16809984h^3+6304p^3+100q^3+25r^3)/(147456Hh^2EF),$$
$$\Delta_4=P(22464a^3+900f^3+195840g^3+22708224h^3+6304p^3+100q^3+25r^3)/(147456Hh^2EF),\ ...$$

To identify the regularity of the formation of coefficients, it was necessary to solve the problem for only four trusses. The following coefficients are obtained using the methods of the Maple system:

$$C_1 = (59 - 20(-1)^n)/256,$$
$$C_2 = 25/147456,\ C_3 = 25/4096,$$
$$C_4 = (43n + 8 - 10(-1)^n)/128,$$
$$C_5 = 2(22n - 10 - (-1)^n),$$
$$C_6 = 197/4608,\ C_7 = 25/36864.$$

The obtained solution differs from the results for planar regular trusses [13], [19], [27], [28] by the presence of constant coefficients C_2, C_3, C_6, C_7. This is because the construction under consideration does not fully satisfy the regularity condition. A pyramidal base with a height of *kh* with an increase in *k* gives additional terms in the solution. That is why the problem is solved at a constant k, for example, the selected three.

Figure 5 shows the curves of the obtained dependence for the dimensionless relative deflection $\Delta' = EF\Delta/(PH)$ at $h_1 = h$, $h = H/(n+3.5)$.

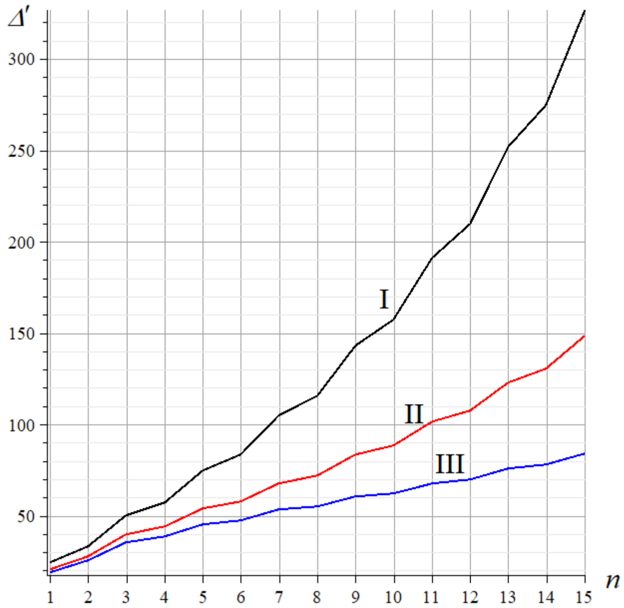

Fig. 5. Dependence of the cantilever deflection on the number of panels,
$H = 25\text{m},\ t = a/4,\ s = a - 2nc,\ \ \text{I} - a = 12\text{m},\ \text{II} - a = 8\text{m},\ \text{III} - a = 5\text{m}$

The analytical form of the solution by Maple methods allows us to reveal the cubic nature of the growth of the deflection $\lim_{n\to\infty} \Delta'/n^3 = 43a^3/(128H^3)$. The uneven growth of the deflection is explained by the presence of terms with a factor of $(-1)^n$ in the solution.

3.1.2 Horizontal displacement of the vertex from the action of the vertical load on the console

Under the action of vertical forces at the end of the console, similar to the off-center compression of the rod in the truss, there is a general curvature of the structure, which can be traced by the horizontal displacement of the top. The solution to the problem of vertex displacement is also obtained by the formula (1) differs from (2) only by the denominator and has the form

$$\Delta = P(C_1a^3 + C_2r^3 + C_3f^3 + C_4g^3 + C_5h^3 + C_6p^3 + C_7q^3)/(EFah). \quad (4)$$

The coefficients in this expression are obtained by induction and have the form

$$C_1=(6n+22-11(-1)^n)/64,$$
$$C_2=(6n+25)/73728,$$
$$C_3=(6n+25)/2048,$$
$$C_4=(3n^2-2(-1)^n+11n+1/2)/16,$$
$$C_5=18n^2+2((-1)^n+29)n+3(-1)^n-23,$$
$$C_7=(41+33n/2)/576,\ C_7=(6n+25)/18432. \tag{5}$$

The figure 6 shows the change in the relative offset from the number of panels for different sizes of the base a. The geometric data of the mast in this calculation is taken the same as in the previous solution. The intersections of the curves are characteristic, illustrating the noticeable non-linearity of the problem. As in the previous problem, the growth of the value is cubic. This shows the following limit: $\lim_{n\to\infty}\Delta'/n^3=3a^2/(16H^2)$.

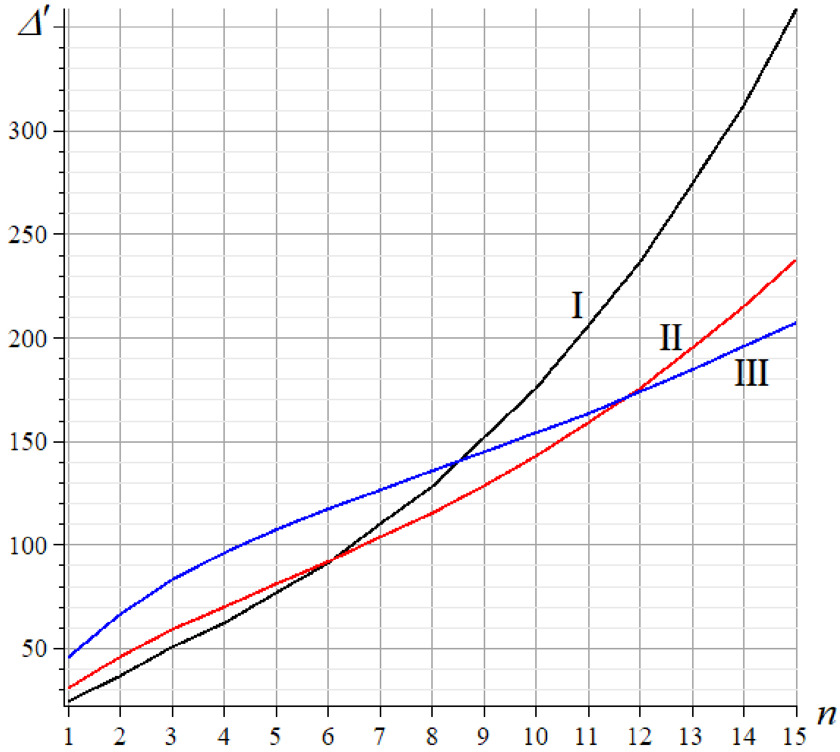

Fig. 6. Dependence of the horizontal deviation of the mast top on the number of panels under the action of a vertical load on the console (Fig. 1), $\mathrm{I}-a=12\mathrm{m},\ \mathrm{II}-a=8\mathrm{m},\ \mathrm{III}-a=5\mathrm{m}$

3.1.3 Horizontal displacement of the vertex from the action of the wind load

The actual operational characteristic of the mast is its resistance to the horizontal lateral load caused by the wind. You can estimate the rigidity of the truss by the displacement of its top in the direction of the wind. The simplest, but not the most accurate model of wind load is the uniform distribution of forces along the vertices of the structure along one windward face of the mast (Fig. 2). The displacement of the vertex under the action of the load is calculated using the Maxwell-Mohr formula (1). The desired formula here has eight constants:

$$\Delta=P(C_1a^3+C_2r^3+C_3f^3+C_4g^3+C_5h^3+C_6p^3+C_7q^3+C_8e^3)/(EFa^2), \tag{6}$$

where $e=\sqrt{2a^2+16h^2}$. The coefficients are obtained by induction based on the results of the calculation of 16 trusses with a consistently increasing number of panels *n*. The obtained solutions were used to determine the sequences of coefficients and the recurrent equations that they satisfied. To find solutions to recurrent equations, the *rsolve* operator of the Maple system was used. The most complex recurrent equation was obtained when deriving the formula for the coefficient C_5:

$$C_{5,n} = 2C_{5,n-1} + 2C_{5,n-2} - 6C_{5,n-3} + 6C_{5,n-5} - 2C_{5,n-6} - 2C_{5,n-7} + C_{5,n-8}.$$

As a result, the desired dependencies of the coefficients on the number *n* of panels are obtained:

$$C_1 = (4n^3 + 2(14-(-1)^n)n^2 + 4(26-3(-1)^n)n - 16(-1)^n + 189)/32,$$

$$C_2 = (2n^3 + 11n^2 + 63n + 345/2)/18432,\ C_3 = (4n^3 + 22n^2 + 146n + 351)/1024,$$

$$C_4 = (4n^4 + 28n^3 + 105n^2 + 190n + 2 - 2(-1)^n)/16,$$

$$C_5 = 4(15n^4 + 82n^3 + (3(-1)^n + 264)n^2 + (18(-1)^n + 476)n + 24(-1)^n - 4)/3,$$

$$C_6 = (44n^3 + 260n^2 + 646n + 743)/1152,$$

$$C_7 = (69n + 2n^3 + 11n^2 + 387/2)/4608,\ C_8 = 1/32.$$

Figure 7 at $k = 3$, $h_1 = h$, $h = H/(n+k+0.5)$ shows the curves of the found dependence in terms of the dimensionless relative deflection

$$\Delta' = EF\Delta/(P_s H),$$

where $P_s = P(2n+7)$ is the total horizontal wind load on the structure.

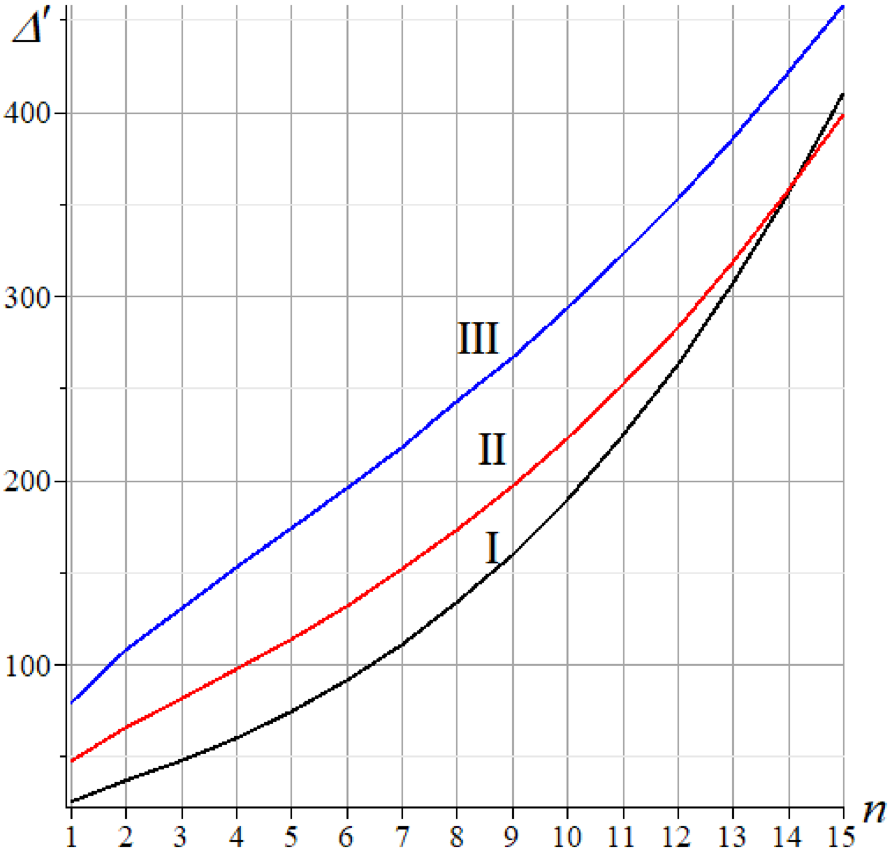

Fig. 7. Dependence of the relative deflection on the number of panels,
$\mathrm{I} - a = 12\mathrm{m},\ \mathrm{II} - a = 8\mathrm{m},\ \mathrm{III} - a = 6\mathrm{m}$

The asymptotics of the obtained dependence turns out to be cubic: $\lim_{n\to\infty} \Delta'/n^3 = a/(8H)$. The nonlinearity of the solution (6) is also shown by the intersection of the curves obtained for different sizes of the base *a*. Here, at *n*=14, the horizontal deviation of the mast top is the same at *a*= 12 m and *a*= 8 m.

3.2 Calculation of forces

To calculate the strength and stability of the structure, you need information about the distribution of forces in the rods. In Figures 7 and 8, the Maple graphical tools show the distribution of the mast rods in the case of vertical load on the console and wind load. Stretched rods are indicated in red, compressed rods are indicated in blue, and unstrained rods are indicated in black. The thickness of the lines is approximately proportional to the modules of the corresponding forces. The numbers show the values of the forces in the rods, related to the value of *P*. The most loaded rods of the mast are not at the bottom of the base, but at the top of it. The alternating height of the compressed and stretched rods in the belts is also characteristic.

For the forces in the vertical rods of the ribs of the upper belt of the structure from the action of the wind load (Fig. 8), the formulas are obtained by induction. Windward force in the rack I:

$$S_{\mathrm{I}} = -P((2n^2+18n+16)hh_1(-1)^n + 3h^2 + h_1(27h+4h_1)(-1)^n)/(ah).$$

The force sign depends on the parity of n and the ratio of h and h_1. When *n*=3, the upwind struts are compressed. Downwind force in rack II:

$$S_{\mathrm{II}} = P((2hn^2+2hn-19h+4h_1)h_1(-1)^n + h(3h+2h_1))/(ah).$$

When $h = h_1$ these expressions are simplified (Fig.2)

$$S_{\mathrm{I}} = -Ph((2n^2+18n+47)(-1)^n+3)/a,$$

$$S_{\mathrm{II}} = Ph(2n^2+2n-15)(-1)^n+5)/a.$$

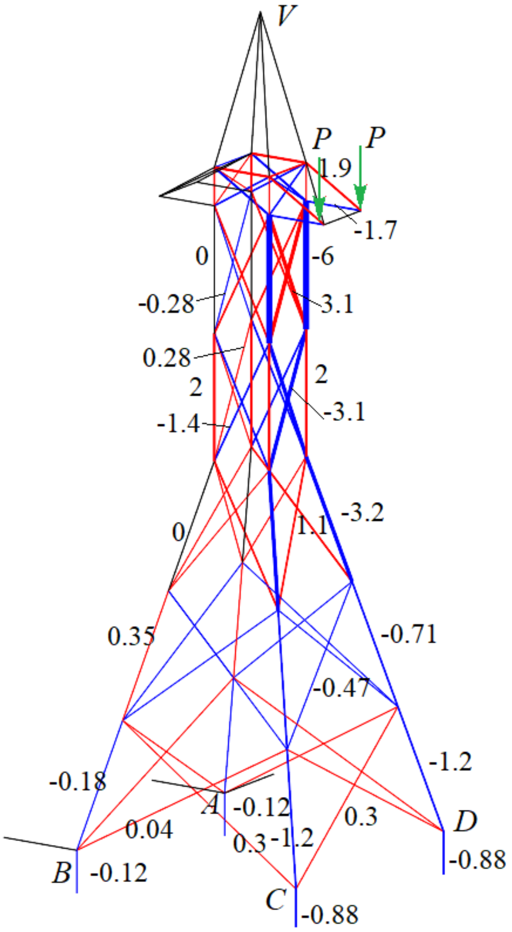

Fig. 7. Distribution of forces on the mast rods from the action of the vertical load on the console,
$a = 6\,\mathrm{m},\ c = a/8,\ h = 3\,\mathrm{m},\ h_1 = 0.9\,\mathrm{m}, k = n = 3$

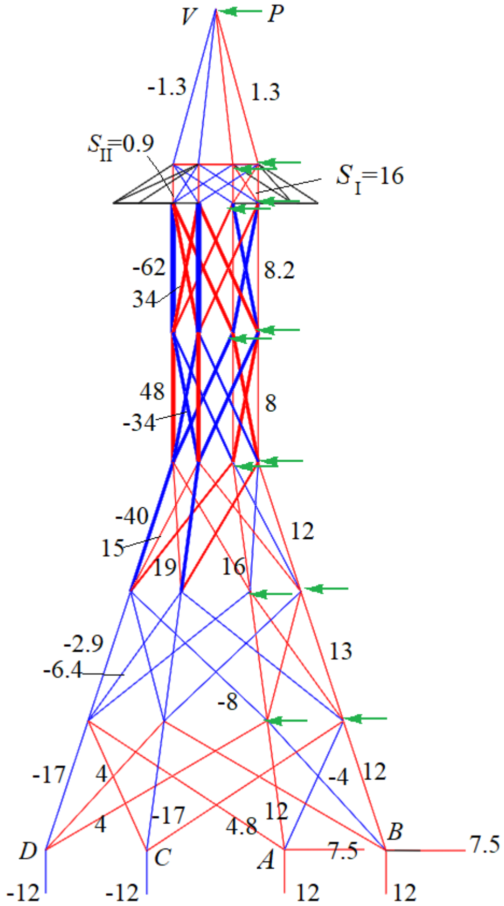

Fig. 8. Distribution of forces on the tower rods from the action of the wind load,
$a = 6\,\mathrm{m},\ c = a/8,\ h = 3\,\mathrm{m},$
$h_1 = 0.9\,\mathrm{m}, k = n = 3.$

4 Conclusions

The main results of the work are as follows.

1. A mathematical model of a statically definable spatial truss of a power transmission line support is proposed. The forces in some structural rods are determined in an analytical form. Patterns of force distribution in all truss rods are constructed.

2.Calculated formulas for the deflection of the console under the action of vertical forces for an arbitrary number of panels and the dependence of the displacement of the mast top on the wind load are derived.

3. Cubic asymptotics of solutions are found.

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