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Deformations and Natural Frequency of a Triangular Truss: Analytical Solutions

Kirsanov, Mikhail Nikolaevich[1]

[1] Moscow Power Engineering Institute, Moscow, Russian Federation; c216@mail.ru;
Correspondence:* email c216@ya.ru; contact phone +79651833534

Keywords:

Triangular truss; Vibrations frequency; Maple; Analytical solution; Deflection; Induction; Dunkerley method; Asymptotics; Maxwell-Mohr formula

Abstract:

The object of research. A new scheme of a lattice externally statically indeterminate truss in the form of a triangle with a lower chord supported by vertical posts and a fixed hinge is considered. When looking for the forces in critical bars and deflection, a vertical load is considered, which is evenly distributed over all external and internal nodes of the truss. The dependence of the deflection of the truss top on the load, dimensions and number of truss panels is given. A formula is derived for the lower estimate of the first frequency of natural oscillations. **Method.** The calculation of forces is carried out by cutting out all the nodes of the structure. The number of unknowns of the system of linear equilibrium equations in the projection on the coordinate axes includes both forces and reactions of supports. The deflection is calculated in analytical form using the Maxwell-Mohr formula and is generalized by induction from solving a number of problems for trusses with a different number of panels to an arbitrary order of a regular truss. To find an analytical estimate of the first frequency of natural oscillations of nodes endowed with masses, each of which has two degrees of freedom, the Dunkerley lower estimate method is used. **Results.** The formulas obtained for the forces in the rods, deflection and the first frequency have a compact form, which can be used to obtain simple evaluation solutions. The lower analytical estimate of the first oscillation frequency is in good agreement with the numerical solution for the entire spectrum of structure oscillations. All necessary transformations are performed in the Maple symbolic mathematics system. Linear asymptotics of solutions for deflection and forces are found.

1 Introduction

Truss structures are used both in construction and in mechanical engineering. The advantage of trusses is their strength, light weight, ease of installation and comparative ease of calculation. Such constructions are usually durable, easy to use. The methodology for calculating the strength, stability and oscillations of real trusses is traditionally based on numerical calculations, usually using the finite element method with specialized packages [1]–[5]. Analytical solutions are used less often. For the first time, Hutchinson R., Fleck N., and Zok F., Latture R, Bergley M. [6]–[8] took up the problem of the existence and calculation of statically determinate regular systems (planar and spatial) that allow analytical solutions for an arbitrary number of panels. Some analytical solutions are known in the form of finite formulas for deformations of regular planar [9]–[12] and three-dimensional trusses [13]. In [14] formulas for the lower estimate of the first natural frequency of planar trusses obtained by induction for an arbitrary order of a regular structure are given. There are another directions in the analytical study of structures using the representation of the solution in the form of trigonometric series [15]–[19] or using the Bubnov - Galerkin method [20]. In [21]–[23], regular constructions are studied in connection with optimization problems.

Handbook [24] contain diagrams of planar statically determinate regular trusses and formulas for calculating their deflection under the action of concentrated and distributed loads of various types.

Here, too, the method of induction was used, which consists in generalizing a series of separate calculations of trusses with a successively increasing number of panels to the case of an arbitrary number of panels. The operators of the Maple symbolic mathematics system were used in analytical calculations. The method of induction is also applicable for deriving the dependences of deformations of spatial trusses [25] on the order of the system (the number of panels).

In this paper, a new scheme of a regular statically definable planar lattice in the form of a triangular truss supported on base nodes is studied. The task was set to derive analytical dependences of the top deflection and oscillation frequency on the number of panels. The results of the performed research can be used in optimization problems and for evaluating numerical solutions, for which, in the case of a large number of rods, numerical calculations may inevitably lead to errors due to the accumulation of rounding errors.

2 Materials and methods

A truss with a length of $2na$ and a height of nh contains $(n+2)(n+3)/2$ nodes, including the support nodes. Number of internal nodes $K=(n+1)(n+2)/2$. The number of bars, including the bars modeling supports, is twice as many as $N=(n+1)(n+2)$. The truss is statically determinate. When calculating the oscillation frequencies of the structure, it is assumed that the mass of the truss is concentrated in nodes that have two degrees of freedom each.

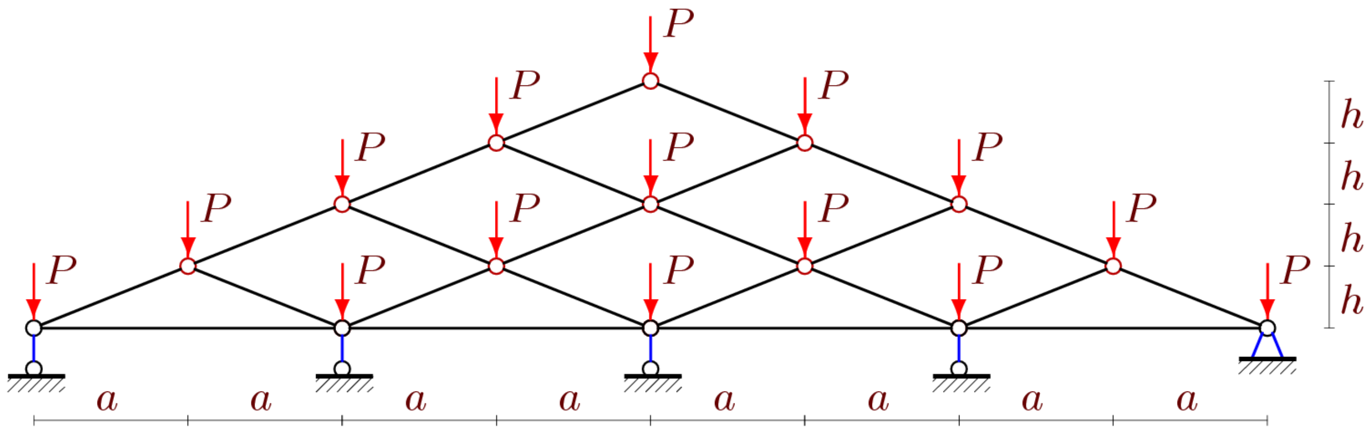

Fig. 1. Truss structure under uniform load n = 4

Forces are calculated in the Maple system using the [26] program. Truss nodes and rods are numbered (Fig. 2). The origin of coordinates is in the left support. Coordinates are set in cycles. Here is the corresponding fragment of the program in the Maple language:

```
s:=0: n1:=(n+1)*(n+2)/2;
for j to n+1 do
  for i to n+2-j do
    x[i+s]:=2*a*i+a*j-3*a:
    y[i+s]:=h*(j-1):
  end:
  s:=s+n+2-j;
end:
for i to n-1 do x[i+n1]:=2*a*i:y[i+n1]:=-h:end:
```

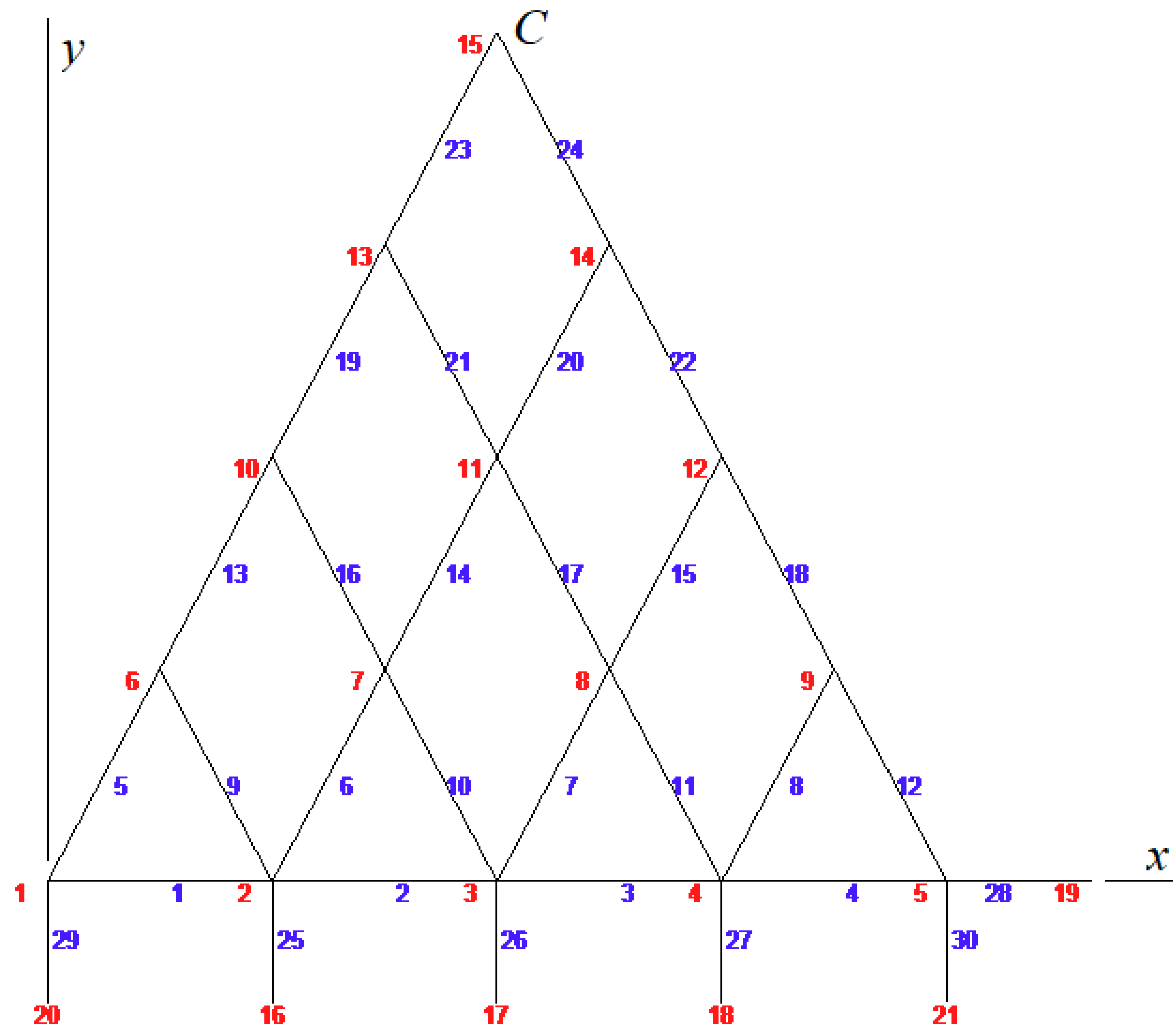

Fig. 2. Numbering of nodes and rods, *n*=4

The structure of the lattice is established by the order of connection of the rods at the nodes. To do this, special lists are introduced $\Phi_\alpha = [i_1, i_2]$ of numbers i_1, i_2 rod ends $\alpha = 1,.., N$. The bars of the lower outer contour, for example, have the following node numbers at the ends: $\Phi_i = [i, i+1],\ i = 1,.., n$. In the same way, the numbers of the ends of the remaining bars of the lattice are set. The system of equilibrium equations of nodes in projections on the coordinate axes is compiled in matrix form $\mathbf{GS} = \mathbf{B}$, where $\mathbf{G}$ — is the matrix of coefficients of the projection equations, $\mathbf{S}$ — is the vector of all forces and reactions of the supports, $\mathbf{B}$ — is the load vector. The projections of the conditional vectors of the rods on the coordinate axes have the form $l_{x,i} = x_{\Phi_{i,1}} - x_{\Phi_{i,2}},\ l_{y,i} = y_{\Phi_{i,1}} - y_{\Phi_{i,2}}$. The matrix $\mathbf{G}$ consists of the direction cosines of the forces. In this case, the same force is applied to different ends of the rod and directed in different directions:

$$G_{2\Phi_{i,2}-1,i} = -l_{x,i} / l_i, G_{2\Phi_{i,2},i} = -l_{y,i} / l_i,$$
$$G_{2\Phi_{i,1}-1,i} = l_{x,i} / l_i, G_{2\Phi_{i,1},i} = l_{y,i} / l_i.$$

where $l_i = \sqrt{l_{x,i}^2 + l_{y,i}^2}$ — rod length.

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The solution of the system of equations can be obtained in symbolic or numerical form.

3 Results

3.1 Forces in rods

The case of loading a truss with a vertical nodal load, uniform across all nodes, is considered (Fig. 1). The load vector on the right side of the equilibrium equation system in this case has the form: $B_{2i} = -P, \quad i = 1,..,K.$ Vertical external forces are written in even lines of this vector, horizontal forces are written in odd lines. Figure 3 shows a picture of the distribution of forces in the bars of the structure. The force values are related to the value *P* of the load on the node and rounded to two significant figures. The most stretched rod was, as expected, in the middle of the lower chord, and the most compressed, in the lower rod of the lateral side.

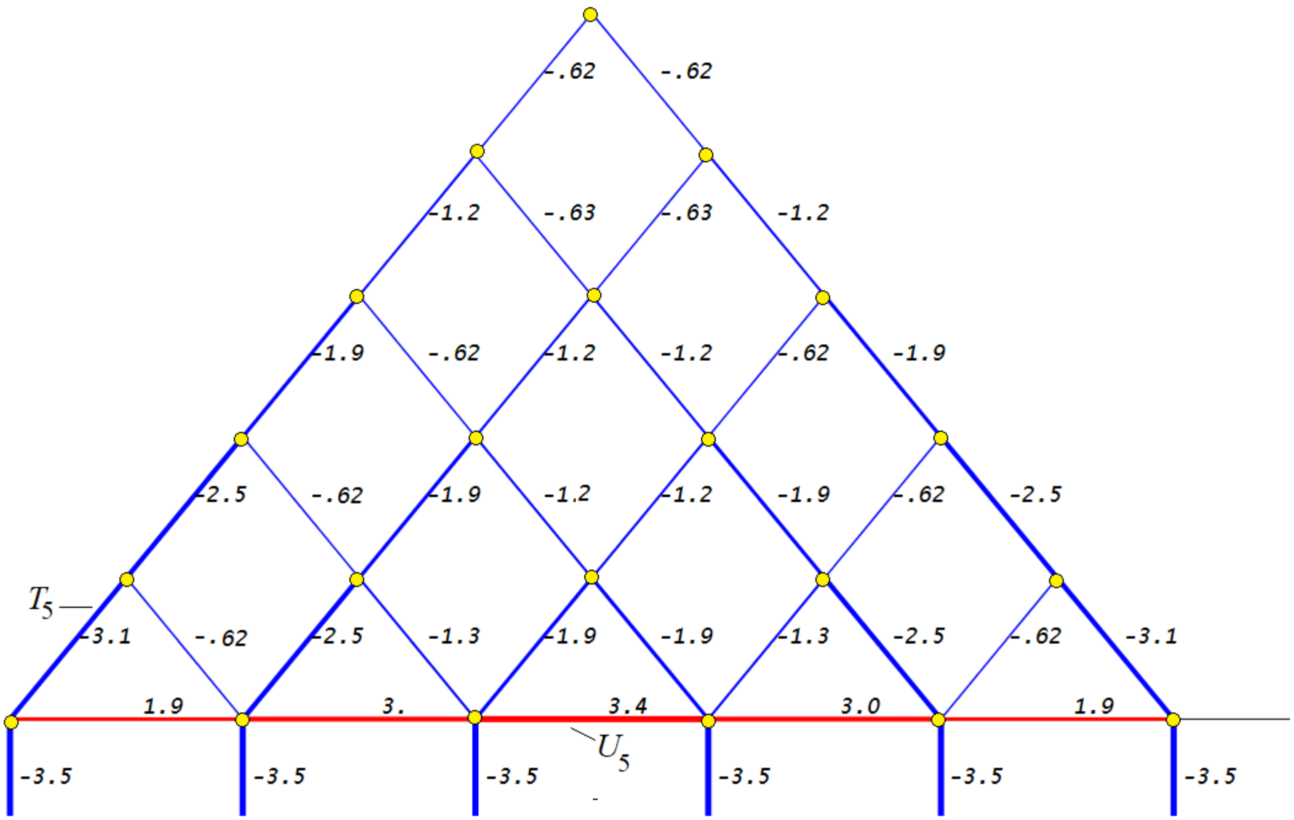

Fig. 3. Distribution of forces in the truss rods, *n*=5, *a*=3 m, *h*= 4 m

The reactions of all vertical supports under such a load are the same and equal to $R = P(n+2)/2$. The red color marks the tensioned rods with positive forces, the blue color — the compressed rods with negative forces. The thicknesses of the segments of the rods are conditionally proportional to the moduli of forces in these rods.

Analytical dependencies on the number of panels of force T_n of the most compressed rod and U_n of the most stretched rod can be done by induction. The sequence of efforts U_n for different *n* has the form $(2h),\ 3aP/h,\ 4aP/h,\ 6aP/h,\ aP/h,\ 3aP/\ 15aP/(2h),\ 10aP/h,\ 12aP/h,\dots$. The recursive equation, which is satisfied by the common term of this sequence, gives the operator rgf_findrecur of the Maple system: $U_n = 2U_{n-1} - 2U_{n-3} + U_{n-4}$. The solution to this equation has the form: $U_n = aP(3(-1)^n + 2n^2 + 4n - 3)/(16h)$. Similarly, the expression for the most compressed rod is we obtained: $T_n = -cPn/(2h)$. These solutions have asymptotic approximations. Let the height of the truss be *H* and the length of the base *L*. The following limits are found

$$\lim_{n\to\infty} U_n / n^2 = PL/(16H), \ \lim_{n\to\infty} T_n / n = -P\sqrt{4H^2 + L^2}/(4H),$$

where $H = bh, L = 2na$.

3.2 Deflection

To calculate the deflection of the top of the truss, the Maxwell-Mohr's formula is used, assuming that all the bars are elastic and have the same stiffness *EF*:

$$\Delta = \sum_{\alpha=1}^{N} S_\alpha^{(P)} S_\alpha^{(1)} l_\alpha / \left(EF\right).$$

The sum is compiled for all elastic bars of the structure, including bars that model supports. The following designations are used in the formula: $S_\alpha^{(P)}$ — is the force in the rod with number α from the action of an external load, $S_\alpha^{(1)}$ is the force in the same rod from the action of a single vertical force applied to the vertex *C*, the deflection (vertical displacement) of which is measured, l_α is the length of the rod.

To derive the formula for the dependence of the deflection on the number of panels, the induction method is used. Calculating the deflection sequentially for trusses of the order *n* = 1, 2, 3, ... , gives:

$$\Delta_1 = P(a^3 + c^3 + 3h^3)/(2h^2 EF),$$
$$\Delta_2 = P(4a^3 + 3c^3 + 4h^3)/(2h^2 EF),$$
$$\Delta_3 = P(10a^3 + 6c^3 + 5h^3)/(2h^2 EF),$$
$$\Delta_4 = P(10a^3 + 5c^3 + 3h^3)/(h^2 EF),$$
$$\Delta_5 = P(35a^3 + 15c^3 + 7h^3)/(2h^2 EF), \ldots$$

where $c = \sqrt{a^2 + h^2}$. Generalizing these formulas using the operators of the Maple system to an arbitrary number *n* gives the following result:

$$\Delta_n = P(C_1 a^3 + C_2 c^3 + C_3 h^3)/(h^2 EF),$$

where the coefficients are found from the solution of homogeneous linear recurrent equations and have the form $C_1 = n(n+1)(n+2)/12, C_2 = n(n+1)/4,\ C_3 = (n+2)/2$.

Similarly, but even simpler, the dependence of the deflection is obtained when a single force applied to the vertex acts on the truss: $\Delta_n = P(n(a^3 + c^3) + h^3)/(2h^2 EF)$.

This solution is much simpler due to the type of stress state of the truss under the action of a concentrated load. Only the lateral rods are compressed, and, moreover, by the same forces, while the rods of the lower chord are also stretched by the same forces. The forces in all other rods under such a load are equal to zero.

Let Δ' the value of the dimensionless deflection, related to the length $L = 2na = 100\text{m}$ of the lateral side of the truss and the total load $P_0 = K\,P$: $\Delta' = EF\Delta_n / (P_0 L)$. The found dependence (4) has a limiting value on the horizontal asymptote $\lim_{n\to\infty} \Delta' = h/(2L)$ (Fig. 4).

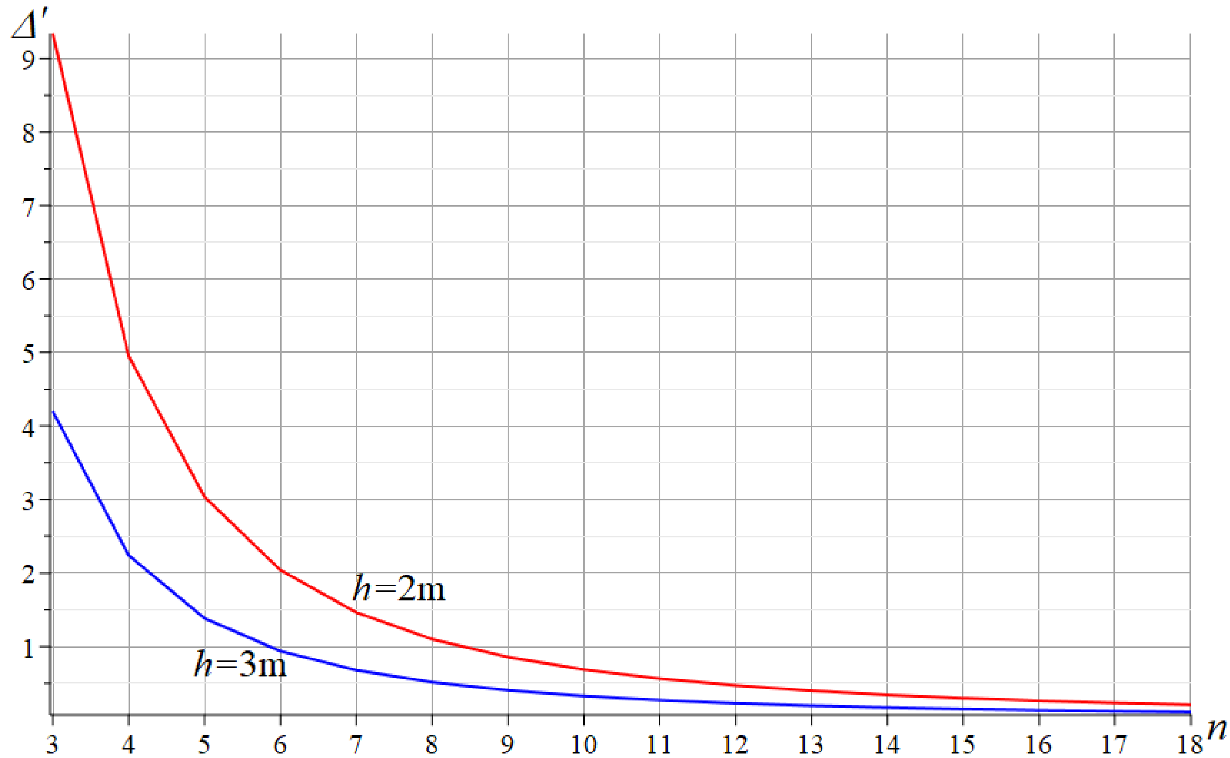

Fig. 4. Relative deflection of the top under the action of a distributed load

3.3 Natural frequency

The value of the first (lowest) frequency of natural vibrations is included in most solutions to problems of structural dynamics. This value is also required to assess the seismic characteristics of the structure [27]–[29]. The lower bound of the first frequency for regular constructions can be obtained in analytical form depending on the number of panels [9], [30].

To calculate the natural vibration frequencies of the considered structure, a simplified but widespread model of the inertial properties of the truss was adopted. It is assumed that the truss rods have no mass, and the entire mass is evenly distributed over the nodes. Each mass has two degrees of freedom. Thus, the total number of degrees of freedom is $2K$. The Dunkerley formula [31], [32] for the lower frequency limit is:

$$\omega_D^{-2} = \sum_{p=1}^{2K} \omega_p^{-2}, \tag{1}$$

where ω_p — are the partial frequencies. Partial frequencies in vertical and horizontal oscillations are considered separately

$$\omega_D^{-2} = \omega_{D,y}^{-2} + \omega_{D,x}^{-2} = \sum_{p=1}^{K} (\omega_{p,y}^{-2} + \omega_{p,x}^{-2}). \tag{2}$$

Partial frequencies of vertical mass oscillations are determined from the equation

$$m\ddot{y}_p + D_p y_p = 0,\ p = 1,...,K \quad , \tag{3}$$

Here D_p — stiffness, the reciprocal of compliance $\delta_p = 1/D_p$. Compliance (vertical displacement) can be calculated using the Maxwell-Mohr formula

$$\delta_p = 1/D_p = \sum_{\alpha=1}^{N} \left(S_\alpha^{(p)}\right)^2 l_\alpha / (EF), \tag{4}$$

where $S_{\alpha}^{(p)}$ is the force in the rod with number α from the action of a vertical unit force applied to the node p, where the mass is located. The stiffness factor and the partial frequency depend on the location where the mass is located. For harmonic vibrations $y_p = U_p \sin(\omega t + \varphi)$, from (3) follows $\omega_{p,y} = \sqrt{D_p / m}$. Substituting this relation in (4) gives the following expression for estimating the first frequency only by partial frequencies of vertical oscillations :

$$\omega_{D,y}^{-2} = \sum_{p=1}^{K} \omega_{p,y}^{-2} = m \sum_{p=1}^{K} \delta_p = m\Delta_{y,n}. \tag{5}$$

Sequential calculation of truss frequencies with a different number of panels shows that the coefficient in $\Delta_{y,n}$ (7) for different n has the form:

$$\begin{aligned}
\Delta_{y,1} &= (5a^3 + c^3 + 5h^3)/(2EFh^2),\\
\Delta_{y,2} &= (4a^3 + 4c^3 + 9h^3)/(2EFh^2),\\
\Delta_{y,3} &= (5a^3 + 5c^3 + 7h^3)/(EFh^2),\\
\Delta_{y,4} &= 10(a^3 + c^3 + h^3)/(EFh^2),\\
\Delta_{y,5} &= (35a^3 + 35c^3 + 27h^3)/(2EFh^2),...
\end{aligned}$$

In general:

$$\Delta_{y,n} = \left(C_{1y}(a^3 + c^3) + C_{2y}h^3\right)/\left(EFh^2\right). \tag{6}$$

The coefficients for a^3, c^3 , and h^3, are obtained from the solution of the corresponding recursive equations:

$$C_1 = n(n+1)(n+2)/12, \quad C_2 = (n+4)(n+1)/4. \tag{7}$$

Similarly, in the case of horizontal oscillations

$$\omega_{D,x}^{-2} = \sum_{p=1}^{K} \omega_{p,x}^{-2} = m \sum_{p=1}^{K} \delta_p = m\Delta_{x,n} = m\left(C_{1,x}a^3 + C_{2,x}c^3 + C_{3,x}h^3\right)/\left(EFh^2\right), \tag{8}$$

where

$$\begin{aligned}
C_{1,x} &= (5n+6)(n+2)(n+1)/12,\\
C_{2,x} &= n(n+1)(n+2)/12,\\
C_{3,x} &= n(n+1)/4.
\end{aligned} \tag{9}$$

Thus, from (2), (5-8) follows the expression for the lower estimate of the first frequency

$$EF\omega_D^{-2} = \left(C_{1y}(a^3 + c^3) + C_{2y}h^3\right)/h^2 + \left(C_{1,x}a^3 + C_{2,x}c^3 + C_{3,x}h^3\right)/a^2 \tag{10}$$

with coefficients (6) and (9).

The errors of the resulting estimate can only be estimated from a comparison with the minimum frequency of the entire spectrum of natural frequencies of the structure obtained numerically. This solution reduces to an eigenvalue problem. The system of differential equations of motion of the masses of the structure with the number of degrees of freedom $r = 2K$ is written in matrix form:

$$m\mathbf{I}_r\ddot{\mathbf{U}} + \mathbf{D}_r\mathbf{U} = 0, \tag{11}$$

where $\mathbf{D}_r$ is the truss stiffness matrix, $\mathbf{U}$ is the mass displacement vector (horizontal and vertical), $\mathbf{I}_r$ is the identity matrix. Multiplying the vector equation (11) on the left by the matrix $\mathbf{B}_r$ inverse to matrix $\mathbf{D}_r$ gives the equation

$$m\mathbf{B}_r\ddot{\mathbf{U}} + \mathbf{I}_r\mathbf{U} = 0. \tag{12}$$

For harmonic oscillations with frequency ω, a connection $\ddot{\mathbf{U}} = -\omega^2\mathbf{U}$ is known .

In this case, from (12) it immediately follows: $\mathbf{B}_r\mathbf{U} = \lambda\mathbf{U}$, where $\lambda = 1/(\omega^2 m)$ are the eigenvalues of the matrix $\mathbf{B}_r$, the elements of which are calculated by the Maxwell-Mohr's formula. To find the eigenvalues of a matrix in the Maple system, there is the Eigenvalues operator from the LinearAlgebra package of linear algebra. The solution here can only be found numerically.

Example. A steel truss with masses of $m = 800\,\text{kg}$ in knots has a panel length of $a = 6\,\text{m}$. The stiffness of the rods is $EF = 1.8 \cdot 10^5\,\text{kN}$, the height of the support posts is h. Figure 5 shows the dependences of the first frequency on the number of panels obtained numerically and analytically.

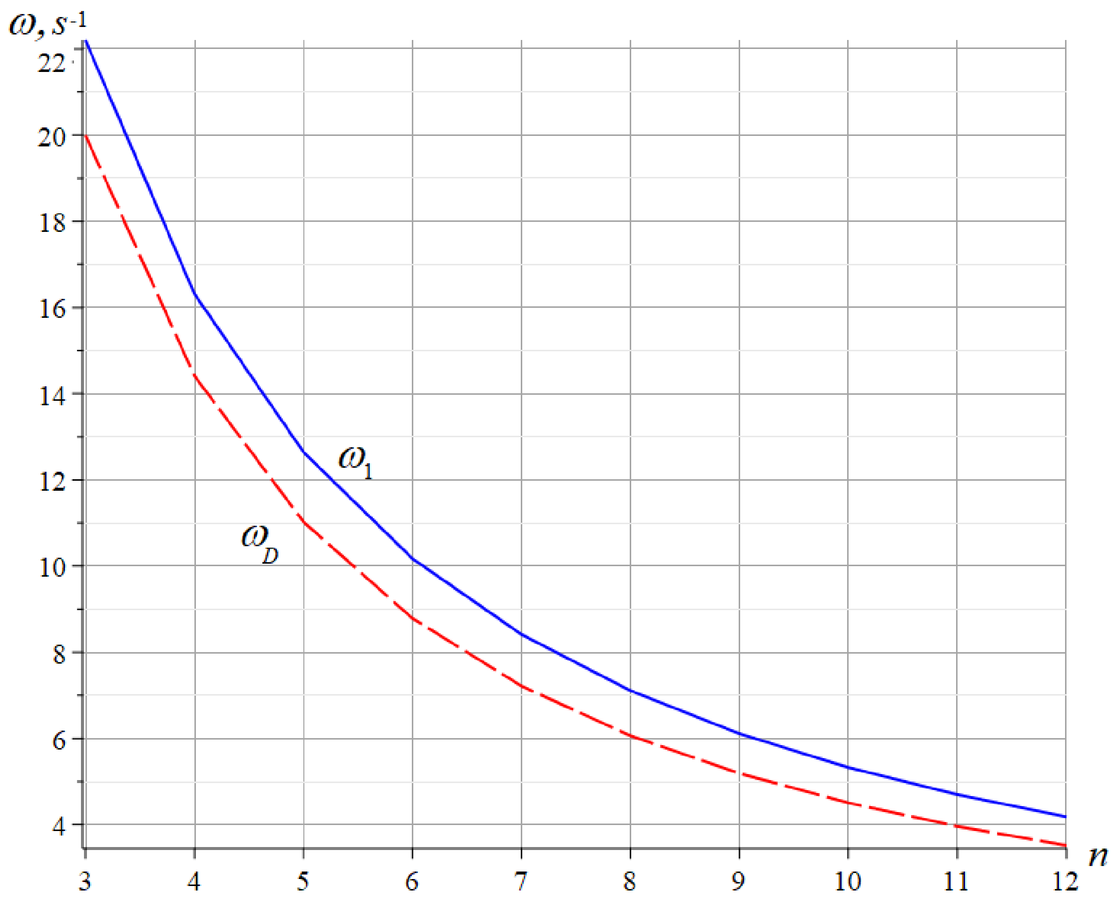

Fig. 5. Comparison of the first frequency ω_1 and ω_D of its lower analytical estimate

The dependence of the relative error $\varepsilon = (\omega_1 - \omega_D)/\omega_1$ on the number of panels (Fig. 6) shows that with an increase in the number of panels, the error increases monotonically, reaching a horizontal asymptote, the value of which depends on the height h.

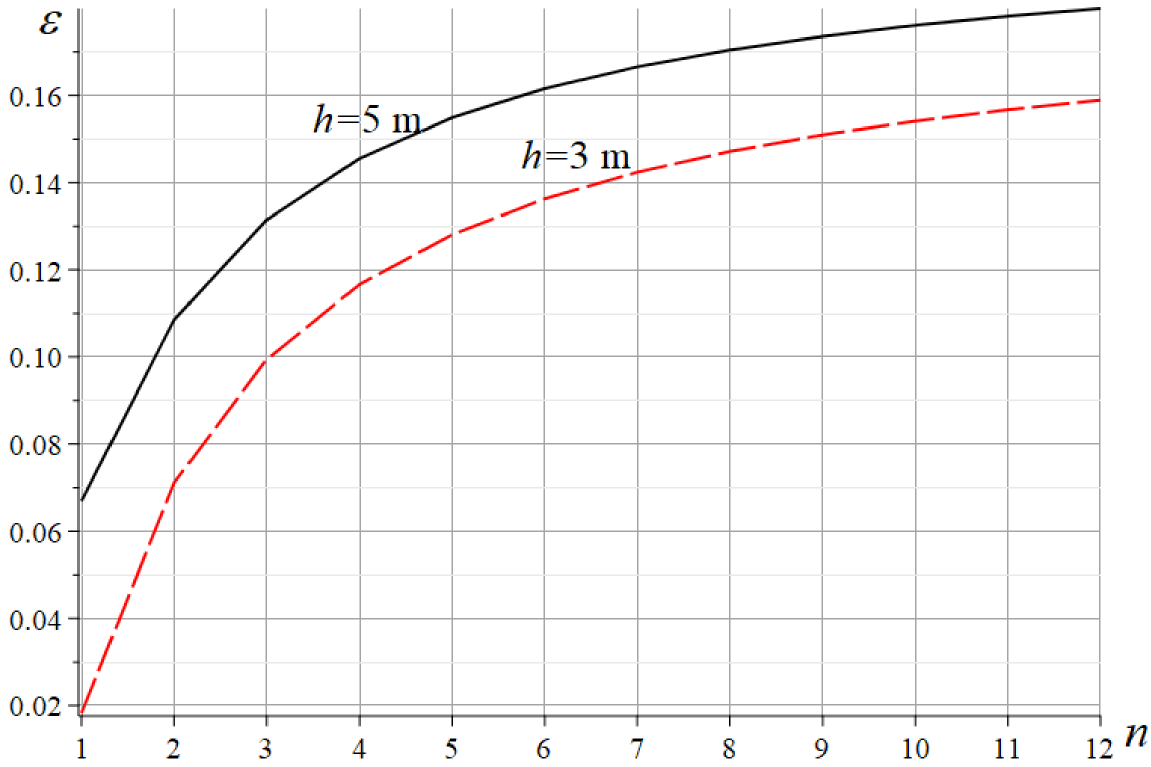

Fig. 6. Error of the analytical solution for natural frequency depending on the number of panels

4 Discussion

A simple but unusual scheme of a rod statically determinate planar structure is proposed. If beam, arch and frame schemes of trusses are well studied and for many of them analytical solutions of problems of both deformations and natural frequencies are found, then for trusses with a triangular outline of the upper chord, such solutions are quite rare [33]. Here, a triangular outwardly statically indeterminate truss with supports located at its base is chosen for analysis. For the method used, external static indeterminacy is not a problem. The system of equilibrium equations for all truss nodes includes not only unknown forces, but also the reactions of the supports. Moreover, the amount of calculations allows not only to obtain an analytical solution, but also to generalize it to an arbitrary number of panels. Such solutions, designed for a wide range of objects under study, are also valuable for solving optimization problems. Separately, it should be noted the features of truss modeling when solving the problem of natural frequency. In most solutions of such problems [30] it is assumed that nodes have one degree of freedom. Horizontal mass movements are usually neglected. This is appropriate for beam schemes of small height, in which, indeed, the horizontal rigidity of the structure is much greater than the vertical one. Trial solutions of the triangular truss problem in such a simplified formulation showed that the accuracy of the Dunkerley estimate is too low. The discrepancy with the numerical solution of the spectrum problem (in the same formulation with vertical mass oscillations) exceeds 50%. Accounting for horizontal mass velocities somewhat complicates solution (11), but a more accurate (also analytical) solution using the Rayleigh energy method is even more complicated and is not presented here.

5 Conclusion

The main results of the work:

1. A new scheme of a symmetric statically determinate triangular truss is proposed.

2. Calculation formulas are found for the forces in characteristic rods and deflection under the action of two types of loads for an arbitrary number of panels. The asymptotics of solutions are revealed.

3. An analytical estimate of the first oscillation frequency of the truss has been obtained. Comparison with the numerical solution with the lowest frequency of the entire spectrum showed good agreement between the solutions.

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