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Formula for the lattice truss fundamental frequency vibration

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Abstract:

The research object is a statically determinate planar lattice truss with an arbitrary number of panels. The design with two hinged supports allows kinematic variability for some numbers of panels regardless of the acting load. **Method**. As applied to a structure with an admissible number of panels, the analytical expression for the first frequency of free vibrations is derived using a simplified version of the approximate Dunkerley method. Comparison of the analytical solution with the numerical one shows its high accuracy, which increases with the number of panels. Analytical transformations were performed in the Maple computer mathematics system. **Results**. The formula for calculating the oscillation frequency has the form of a polynomial in the number of panels and can be used for simple evaluation of solutions obtained numerically. The algorithm for deriving the frequency formula can be used in similar problems of structural mechanics.

1 Introductions

Calculation of natural frequency of vibrations of building structures in engineering practice is usually performed in computer systems based on the finite element method [1]–[3]. Solutions are known in computer mathematics systems using the initial function method, the analytical decomposition method, and trigonometric polynomials [4],[5]. A separate area of analytical calculations of regular rod systems is the induction method [6], [7]. An analytical estimate of the deflection of a rod model of a hipped roof frame was obtained in [8]. Resonance safety zones and an estimate of the first frequency of a statically determinate flat lattice truss with a rise in the middle part were obtained in [9]. The reference books [10], [11] contain formulas for calculating the deflection of flat regular trusses obtained in the Maple computer mathematics system using the induction method. The problem of calculating regular rod structures was apparently first addressed by Hutchinson, R.G. and Fleck, N.A. [12], [13].

The lowest frequency of a cantilever spatial truss was calculated analytically in [14], [15]. The formula for the deflection of a regular flat truss was derived by the induction method in [16]. In [17], formulas were obtained for the deflection of a lattice truss taking into account its kinematic variability for a certain number of panels. A distribution scheme for virtual node velocities was found in the case of structure variability. The analytical method of initial functions for calculating a simply supported rectangular plate was used in [18]. The deflection of a composite truss was calculated analytically in [19]. The formula for the first natural frequency of oscillations of a two-span truss was obtained in [20]. In [21], analytical estimates of the deflection and the first natural frequency of free oscillations of a thrust flat arched truss with a cross-shaped lattice were obtained. The spectra of a family of regular trusses were analyzed, in which spectral constants were found. The formula for the dependence of the deflection of a flat statically determinate truss with an arched lattice on the height and number of panels was derived in [22]. In [23], a low-dimensional, high-precision model of a beam truss with geometric nonlinearity was developed using the reduced-order modeling method. A new approach to constructing a reduced-order model of a planar beam-like periodic truss with high accuracy and high computational efficiency is presented in [24]. In [25], an equivalent nonlinear beam model for analyzing forced vibrations of a beam truss and its numerical implementation for describing the behavior of a periodic structure were developed. A general procedure for determining the equivalent properties of beams of beam-type lattice trusses based on the energy equivalence method was developed in [26]. In [27], a design-oriented study with a simplified theoretical solution using the equivalent beam method was carried out to estimate the bending stiffness of a fiber-reinforced polymer and aluminum structure with spatial decking and a truss. The homogenization concept and the shear equivalence principle were used. An analytical expression for the first natural frequency of a planar truss using the Maple computer mathematics system was obtained in [28]. The static calculation of truss deformations in analytical form is performed in [29]. The formulas for the dependence of the deflection of a trapezoidal beam truss on the number of panels are obtained and analyzed in [30].

There are no simple analytical solutions in the scientific literature for the problem of the natural frequency and frequency spectrum of a double-lattice truss with an arbitrary number of panels. Known numerical solutions for problems posed for similar trusses with simpler lattices do not allow analytical studies of the structure for the purpose of their optimization. In addition, numerical solutions for trusses with a large number of panels tend to lose accuracy and are usually labor-intensive. In this paper, as a continuation of the work [17], an analytical dependence of the first natural frequency of a new flat lattice truss scheme is sought. The problem is expanded to include the analysis of the entire spectrum of natural frequencies of the structure.

2 Materials and Methods

2.1 Truss design

The structure under consideration is a statically determinate flat beam truss with two supports and beveled edges (Fig. 1). The length of the truss with 2n panels in the span is $L = 2an$, the height is 3*h* [17]. The inertial properties of the truss are modeled by concentrated masses distributed over all $K = 4n + 2$ nodes of the structure. The truss consists of $\eta = 8n + 4$ rods. This number also includes three rods modeling the left movable and right fixed supports.

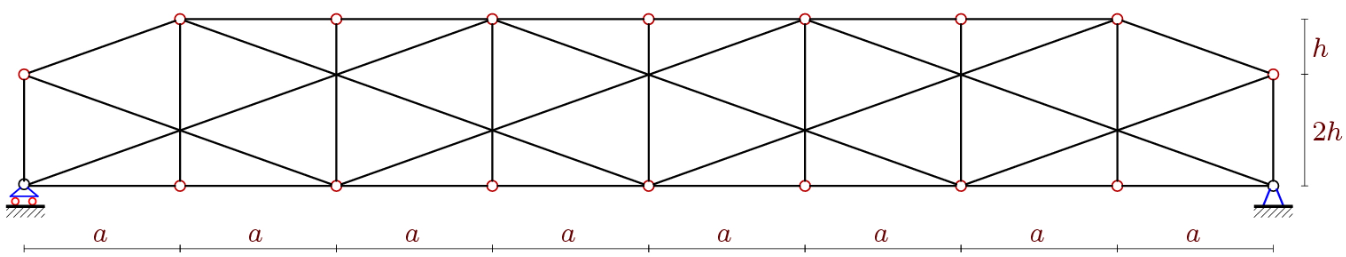

Fig. 1 – Truss, $n = 4$

The calculation of forces in the rods is performed in analytical form using the computer mathematics system Maple [31]. Expressions for forces depending on the current nodal load are calculated by the method of cutting out nodes. The direction cosines of the forces in the rods included in the projection equations are determined by the coordinates of the nodes and the structure of the connection of the rods. The numbering of the rods and nodes of the truss using the example of a truss with five panels in half a span is shown in Figure 2.

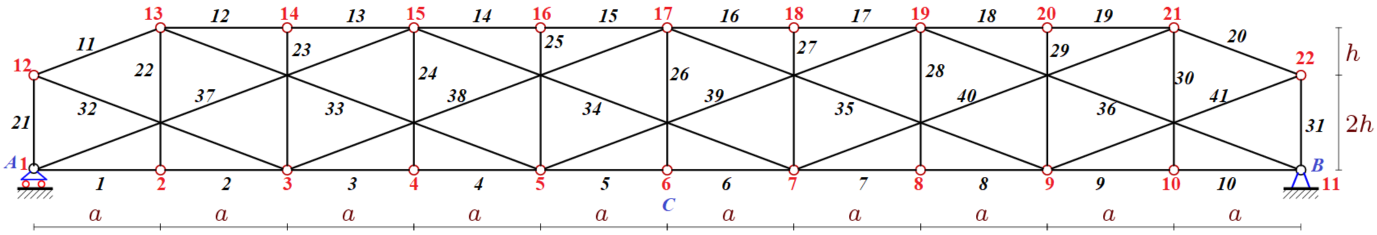

Fig. 2 – Numbering of bars and nodes, $n=5$

The coordinates of the nodes are as follows:

$$x_i = a(i-1),\ y_i = 0,\ i = 1,..,2n+1,$$
$$x_{i+2n+2} = ai,\ y_{i+2n+2} = 3h,\ i = 1,..,2n-1,$$
$$x_{2n+2} = 0,\ y_{2n+2} = 2h,\ x_{4n+2} = 2na,\ y_{4n+2} = 2h.$$

The structure of the connection of individual rods into a lattice is specified by special vectors $\overline{q}_i$, $i = 1,...,\eta$, containing the numbers of the ends of the corresponding rods by analogy with the assignment of a list of edges of a graph in discrete mathematics. The following vectors correspond to the upper and lower chords of the truss:

$$\overline{q}_i = [i, i+1],\ \ \overline{q}_{i+2n} = [i+2n+1, i+2n+2],\ i = 1,...,2n.$$

The lattice (posts and braces) are set as follows:

$$\overline{q}_{i+4n} = [i, i+2n+1],\ i = 1,...,2n+1,$$
$$\overline{q}_{6n+2} = [3, 2n+2],\quad \overline{q}_{8n+1} = [2+4n, 2n-1],$$
$$\overline{q}_{i+6n+2} = [2i+3, 2i+2n+1],\ \overline{q}_{i+7n+1} = [2i+2n+3, 2i-1],\ i = 1,...,n-1.$$

The following vectors correspond to the rods of the side supports:

$$\overline{q}_{\eta-2} = [1, 4n+3],\ \ \overline{q}_{\eta-1} = [2n+1, 4n+4],\ \overline{q}_{\eta} = [2n+1, 4n+5].$$

The coefficients of the system of equations of equilibrium of nodes in projections on the coordinate axes are calculated from these data. The system of equations of equilibrium of nodes in the matrix version has the form:

$$\mathbf{GS} = \mathbf{B} \qquad (1)$$

where $\mathbf{G}$ is the matrix of direction cosines of forces. The vector of loads on nodes is designated as $\mathbf{B}$, the vector of unknown forces and support reactions is $\mathbf{S}$. The length of these vectors is equal to the number of truss nodes *K*. Odd elements of the load vector B_{2i-1} are projections of external forces applied to node *i* on the horizontal x-axis, even elements are projections of forces applied to the truss on the vertical y-axis. The elements of the matrix $\mathbf{B}$ (direction cosines of the force vectors on the coordinate axes) are calculated using data on the lattice structure and node coordinates:

$$l_{x,i} = (x_{q_{i,1}} - x_{q_{i,2}}) / l_i,\ l_{y,i} = (y_{q_{i,1}} - y_{q_{i,2}}) / l_i,\ \ i = 1,...,\eta,$$

where $l_i = \sqrt{l_{x,i}^2 + l_{y,i}^2}$ is the length of the rod with number *i*. In every two consecutive rows of the matrix $\mathbf{B}$, the coefficients of the projection equations on the *x* and *y* axes for one node are written:

$$G_{2\eta_{i,1}-1,i} = l_{x,i} / l_i,\ G_{2\eta_{i,1},i} = l_{y,i} / l_i,$$
$$G_{2\eta_{i,2}-1,i} = -l_{x,i} / l_i, G_{2\eta_{i,2},i} = -l_{y,i} / l_i.$$

In the Maple program, the inverse matrix method is used to find a solution to system (1) in analytical form. In [17], for the truss under consideration, it is shown that for panel numbers multiple of three, the determinant of the matrix $\mathbf{G}$ becomes zero. This indicates the kinematic variability of the structure. If we introduce a sequence of panel numbers with a common term $n = (3(2k-1)-(-1)^k)/4,\ k = 1,2,3,...$, then the corresponding sequence of trusses will not include kinematically variable schemes. For this sequence of trusses, the formula for the first frequency of free oscillations of the structure is derived in this paper.

2.2 Natural oscillation frequency

A model of a truss with equal masses at the nodes is considered. The calculation of the forces from the action of a vertical load *P* uniformly distributed over the nodes at *n* = 4, *a* = 3m, *h* = 2m gives a picture of the distribution of forces in the rods, presented in Figure 3.

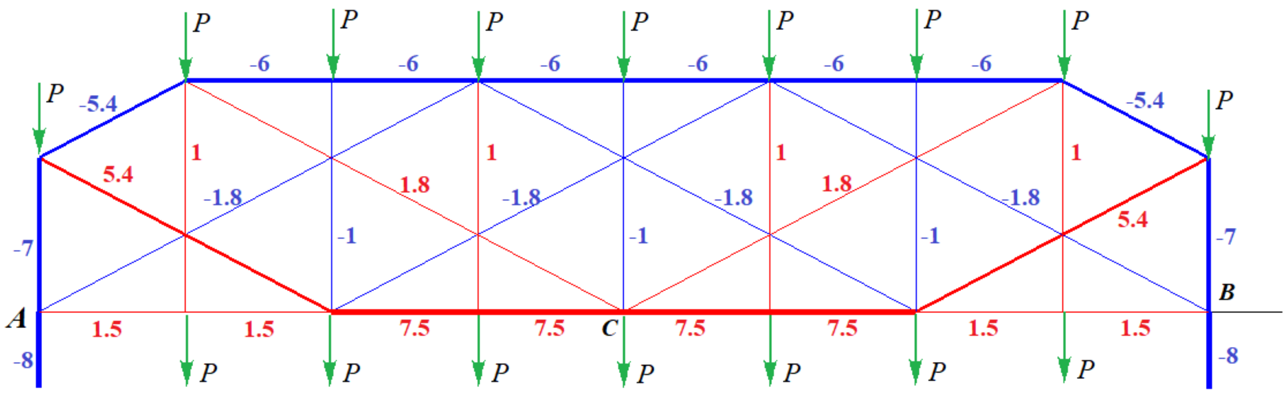

Fig. 3 – Forces in truss rods, $n = 4$

The compressed rods are marked in blue, and the stretched ones in red. The rounded values of the forces are related to the nodal load P. The main load falls on the truss chords and on the lateral descending diagonals. The least loaded are the posts and two lateral rods in the lower chord. In calculating the frequency of natural oscillations, it is assumed that the masses in the nodes perform only vertical movements. In this case, the considered model of the structure has K degrees of freedom. The lower estimate of the first oscillation frequency according to the simplified Dunkerley formula is expressed as follows:

$$\omega_n^{-2} = m\sum_{i=1}^{K}\delta_i = m\delta^{\max}K/2 = m\tilde{\Delta}_n,$$

where $\delta^{\max}$ is the largest value of deflection $\delta_i,\ i = 1,..,K$ due to the action of a vertical unit force on node *i* over all nodes, *m* is the mass at the node. In the truss under consideration, the node with the largest deflection is located in the middle of the lower chord and has the number *n* +1 (Fig. 2). In the original Dunkerley method, the sum of deflections is calculated over all nodes of the truss $\sum_{i=1}^{K}\delta_i$, and in the simplified version, the calculation of the sum is replaced by its average value $\delta^{\max}K/2$. The analytical value $\tilde{\Delta}_n$ is calculated using the Maxwell - Mohr formula as the sum over all rods of the structure, including the three supporting ones:

$$\tilde{\Delta}_n = K\sum_{\alpha=1}^{\eta}\left(S_\alpha^{(p)}\right)^2 l_\alpha/(2EF), \tag{2}$$

where $S_\alpha^{(p)}$ is the value of the force in the rod with number α when a unit vertical force acts on the node with number *p* = *n*+1, l_α is the length of the corresponding rod. The stiffnesses *EF* of all the rods of the truss are considered equal. Calculating the sums in (2) for a sequence of trusses with an increasing number of panels yields the following formulas:

$$\tilde{\Delta}_1 = K\left(a^3 + 3c^3 + 14h^3\right) / (4h^2 EF),$$
$$\tilde{\Delta}_2 = K\left(3a^3 + 3c^3 + 4h^3\right) / (2h^2 EF),$$
$$\tilde{\Delta}_3 = K\left(3a^3 + 3c^3 + 2h^3\right) / (h^2 EF),$$
$$\tilde{\Delta}_4 = K\left(21a^3 + 15c^3 + 14h^3\right) / (4h^2 EF),$$
$$\tilde{\Delta}_5 = K\left(39a^3 + 21c^3 + 14h^3\right) / (4h^2 EF), ...$$

For an arbitrary number of panels, this expression has the form:

$$\tilde{\Delta}_n = K\left(C_1 a^3 + C_2 c^3 + C_3 h^3\right) / (EFh^2),$$

where the coefficients are obtained in the Maple system by induction:

$$C_1 = \left(2k^3 - \left(3 + (-1)^k k\right)k^2 + \left((-1)^k + 27\right)k + 5(-1)^k - 13\right) / 32,$$
$$C_2 = \left(18k - 9 - 3(-1)^k k\right) / 16,$$
$$C_3 = \left(3\cos(k\pi/2) + 3\sin(k\pi/2) + 11\right) / 4.$$

Hence, the calculation formula for the first natural frequency is:

$$\omega^* = h\sqrt{\frac{EF}{mK\left(C_1 a^3 + C_2 c^3 + C_3 h^3\right)}}\ . \quad (3)$$

3 Results and Discussion

The obtained estimate of the dependence of the first natural oscillation frequency on the number of truss panels can be compared with the minimum value of the entire frequency spectrum obtained numerically considering all degrees of freedom of the system. As an example, trusses made of steel rods with an elastic modulus of $E = 2.1 \cdot 10^5$ MPa and a rod cross-sectional area are considered $F = 9\,sm^2$. The masses concentrated at the nodes $m = 200\,kg$, the panel size $a = 3m.$ In Figure 4, the curve ω_1 corresponds to the numerical solution obtained as the minimum natural frequency of the entire spectrum of natural frequencies of the truss.

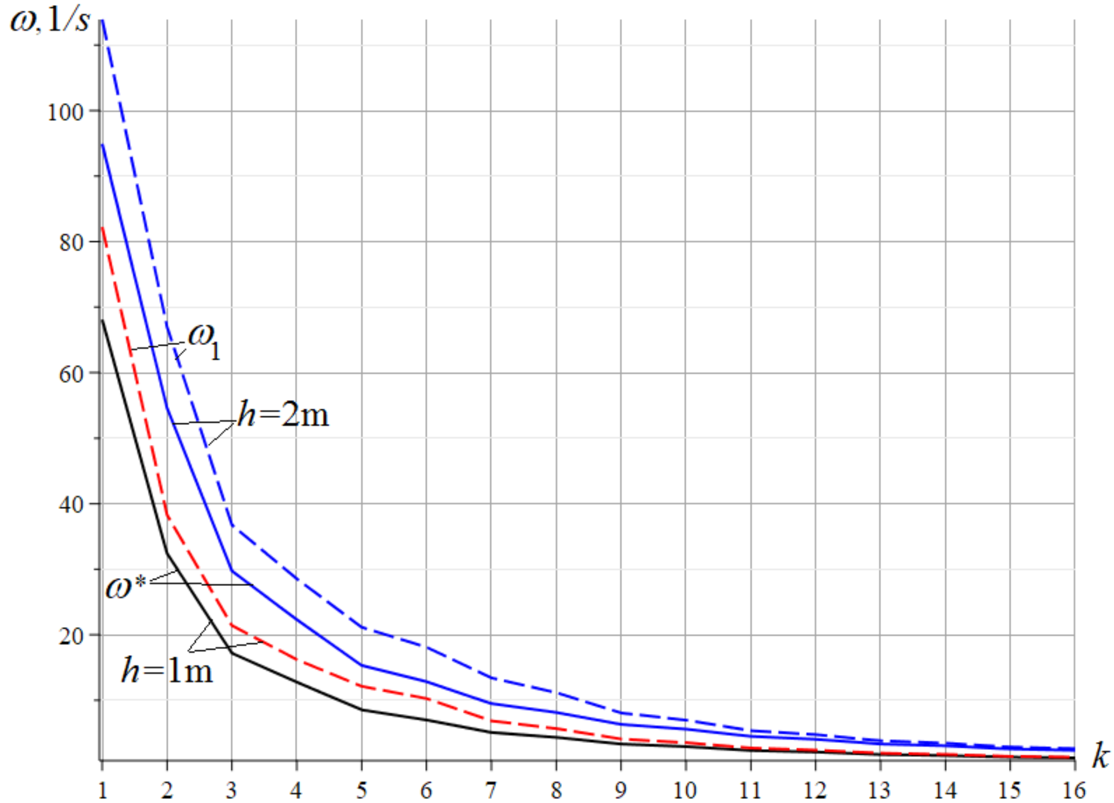

Fig. 4 – Dependence of the first frequency on the number of truss panels

The solution uses the *Eigenvalues* operator for calculating the matrix eigenvalues from the *LinearAlgebra* package of the Maple system. The curve ω^* is plotted using formula (3).

From Figure 4, it is clear that the numerical and analytical methods yield very close results. With an increase in the number of panels, the first natural frequency decreases monotonically.

The obtained analytical solution can be more accurately estimated using the value of the dimensionless relative error $\varepsilon = |\omega_1 - \omega_*|/\omega_1$. The curves in Figure 5, plotted for two values of the truss height, show that starting with *k*=9, the error is within the permissible range and decreases with an increase in the number of panels.

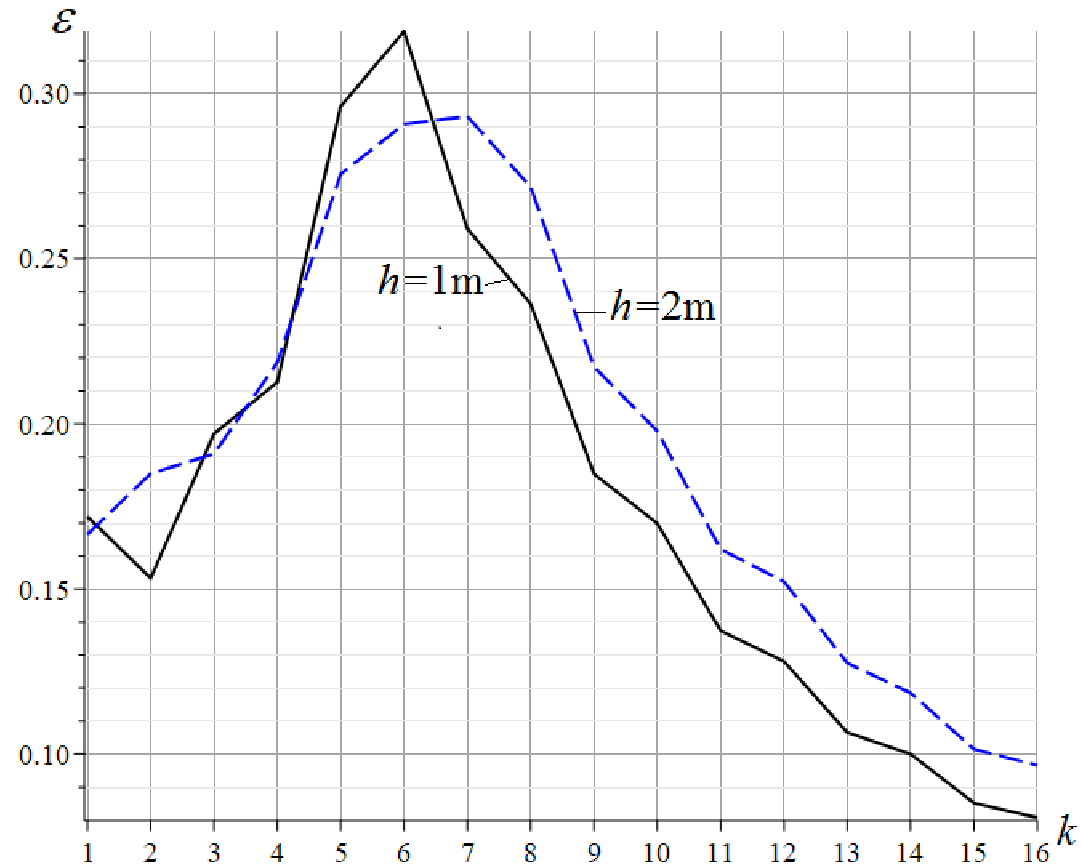

Fig. 5 – Relative errors of methods $a = 3m$

The height of the structure *h* has almost no effect on the accuracy of the analytical solution. A local increase in error to 30% is also noticeable at *k*=6.

3.1 Natural frequency spectra of regular trusses

In practical calculations of the dynamics of structures, the values of the first natural frequencies of natural oscillations are usually the most significant. The entire spectrum of frequencies of a structure is rarely used, and this is usually due to the presence of some external dynamic effects that are potentially dangerous when resonance occurs. The mathematical apparatus of the Maple system, used for the numerical verification of the analytical value of the first natural frequency, can also be used to calculate all frequencies of the structure.

The graph (Fig. 6) shows the spectra of fourteen regular trusses of orders $k = 3,...,16$, which corresponds to the number of panels in half a span *n* from 4 to 23. For trusses of different orders, the frequencies of the spectra are numerically obtained. Each curve corresponds to a truss of a certain order. The ordinates of the points on it are the corresponding natural frequencies. The numbers of natural frequencies in the ordered spectrum are marked with the value *i* on the abscissa axis.

The frequency distribution pattern reveals clear patterns. The most noticeable are the two horizontal lines $\omega_I = 433 s^{-1}$ and $\omega_{II} = 915 s^{-1}$ (spectral constants, [21]). The presence of these constants allows one to predict the frequencies of large-scale trusses with many panels quite simply and with high accuracy based on the calculated data for a truss with a small number of panels. In addition, one notices very significant jumps in the frequency values. The most significant for the selected truss sizes are the jumps from the value $\omega = 200 s^{-1}$ to the first spectral constant ω_I and from ω_I to $\omega \approx 560 s^{-1}$. The largest jump occurs at higher frequencies from $\omega \approx 630 s^{-1}$ to $\omega_{II} = 915 s^{-1}$. It should also be noted that the frequencies of the spectral constants are multiples.

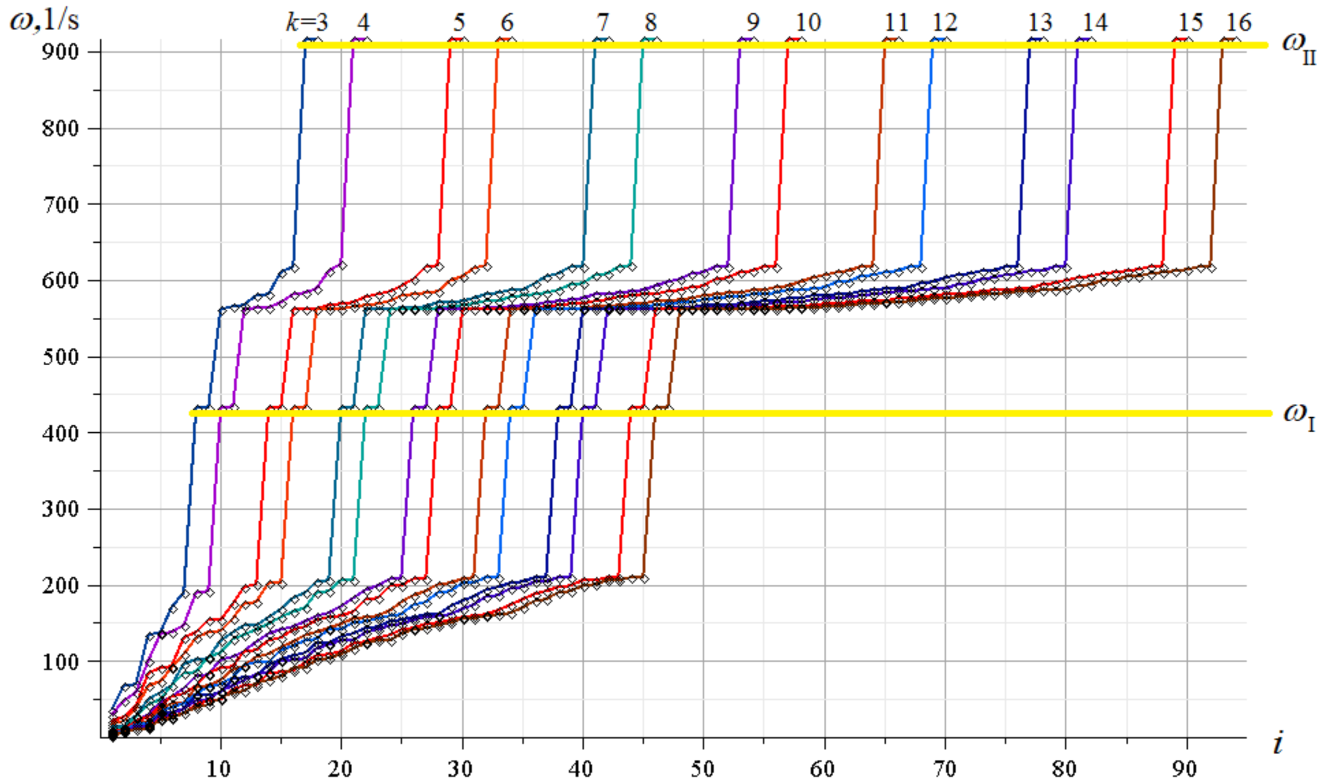

Fig. 6 – Spectra of regular trusses $a = 3m,\ h = 2m.$

4 Conclusions

The induction method with the involvement of computer mathematics system operators was used to derive a formula for the dependence of the first natural frequency of free oscillations of a lattice truss on the number of panels. Earlier, for the same truss scheme in [17], it was noted that for some numbers of panels the scheme kinematically degenerates and analytical solutions for static deflection were obtained. A simplified version of the Dunkerley method was used to determine the natural frequency, which provides good solution accuracy. It was noted that the accuracy of the analytical solution increases with an increase in the number of panels. A pattern of frequency distribution in the spectra of trusses of different orders was constructed and some patterns were found.

The following main conclusions can be made:

1. The estimate of the lowest natural frequency obtained by the simplified method is quite compact and at the same time provides good accuracy, increasing with an increase in the number of panels.
2. Multiple frequencies and spectral constants were noted in the spectrum of natural frequencies of regular trusses of different orders.
3. Resonance safety zones were discovered for natural frequencies.

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