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Analytical calculation of an externally statically indeterminate truss oscillation frequency

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Keywords:

Truss; Kinematic Variability; Computer Mathematics System; Natural frequency; Simplified Dunkerley method; Analytical solution

Abstract:

The research object is a planar braced jointed frame with a cross-shaped double lattice. The mass of the truss is evenly distributed across its nodes. The task is to obtain an analytical dependence of the fundamental frequency of the frame's vibrations on its dimensions, mass, and number of panels. **Method.** The frame is statically determinate. The method of cutting nodes is used to determine the forces in the rods and the reactions at the supports. The stiffness of the structure is calculated using the Maxwell - Mohr formula. The simplification of the frequency calculation for systems with many degrees of freedom is achieved by applying the Dunkerley method. **Results**. Compact formulas for the deflection of the frame and its natural frequency of vibration have been obtained. A case of kinematic variability of the structure with an odd number of panels in half the span was discovered. A comparison of the analytical result with the numerical one shows good accuracy of the proposed method. Some asymptotic solutions have been found.
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1 Introductions

In practice, computer programs based on the finite element method are used to calculate the natural frequency of rod oscillations [1], [2]. Independently of this, analytical methods are also being developed that are free from the shortcomings of numerical methods associated with the inevitable effect of accumulation of rounding errors, which manifests itself in large-scale structures containing a large number of elements. One of the methods for overcoming the "curse of dimensionality" is analytical, based on the use of computer mathematics systems [3], [4] and the induction method of generalizing a number of particular solutions to the general case [5]. Computer mathematics methods are also used to obtain analytical solutions to problems in structural mechanics using the theory of initial functions and functional series [6], [7]. Some of the first to raise the question of the possibility of analytical studies of rod systems of regular structure were, apparently, professors of the University of Cambridge Fleck, N.A. and Hutchinson, R.G. [8], [9]. Among the problems for which analytical solutions were found, problems for planar trusses can be singled out separately. The reference books [5], [10] contain diagrams of planar regular statically determinate trusses and formulas for deflections and natural vibration frequencies depending on the number of panels.

Formulas for the first frequency of a cantilever spatial truss by the induction method were derived in [11], [12]. The vibrations of a rectangular truss with diagonals forming a four-slope symmetrical spatial structure were studied in analytical form by the induction method in [13]. Regular truss schemes in some cases have a hidden and dangerous feature. For some numbers of panels, regardless of the load, such schemes allow kinematic variability [14]. An approximate formula for calculating the fundamental natural

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frequency of oscillations of a two-span truss by the induction method in the Maple system was obtained in [15]. In [16], analytical estimates of the deflection and the first natural frequency of free oscillations of a thrust planar arch truss with a cross-shaped lattice were found. The joint spectrum of a family of regular trusses was also analyzed, where frequencies were found that were the same for trusses of different orders (spectral constants). The dependence of the deflection of a planar statically determinate arch truss on the number of panels was derived in [17]. An analytical expression for the first natural frequency of a planar truss using the Maple computer mathematics system was obtained in [18]. The static calculation of the deflection of planar regular trusses in analytical form is performed in [19], [20]. The formula for the first natural frequency of a planar truss is derived in [21], [22]. A two-sided analytical estimate of the fundamental frequency of oscillations of a flat truss was obtained in [23], [24]. Nonlinear vibrations of rod structures were studied in [25], [26]. In [27] the problem of symmetric free vibrations of pyramidal trusses is solved taking into account the general large measure of deformation (quadratic and logarithmic).

In this paper, a formula is derived for the dependence of the first natural frequency of a new scheme of a planar lattice externally statically indeterminate truss with two fixed supports. An algorithm is proposed that simplifies the form of the solution and gives it a higher accuracy, an analytical solution is found for the deflection of the truss and the asymptotics of this solution.

2 Materials and Methods

2.1 Truss design

The structure under consideration is a statically determinate planar frame truss with two fixed supports and a double diagonal lattice (Fig. 1) [28]. The length of the truss with $2n$ panels in the middle part of the span is $L=2a(n+1)$, the height is $3h$. The mass of the truss is modeled by concentrated loads of mass m, uniformly distributed over all $K=4n+12$ nodes of the structure. The truss consists of $\eta=8n+20$ rods. This number also includes four rods modeling the left and right fixed supports.

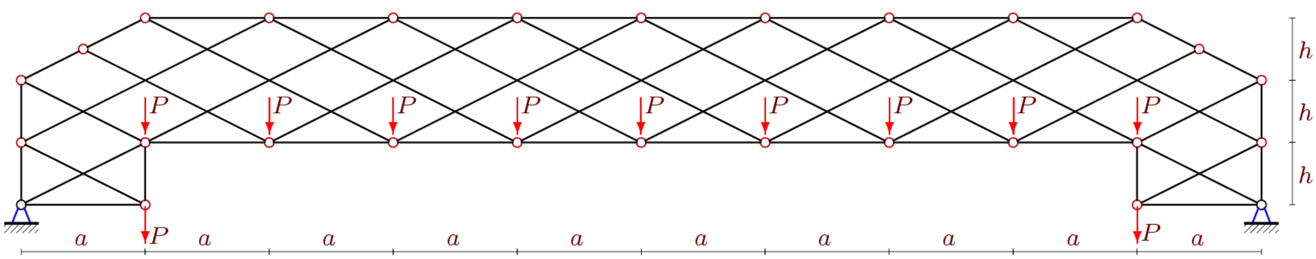

Fig. 1 – Truss, $n=4$

The calculation of forces in the rods is performed in analytical form using the computer mathematics system Maple [29]. The forces in the rods are calculated by cutting out nodes in analytical form. The projections of the forces in the rods onto the coordinate axes are determined by the coordinates of the nodes and the structure of the connection of the rods. The numbering of the rods and nodes of the truss is performed in parametric form depending on the number of panels. The numbering of the nodes for n = 2 is shown in Figure 2.

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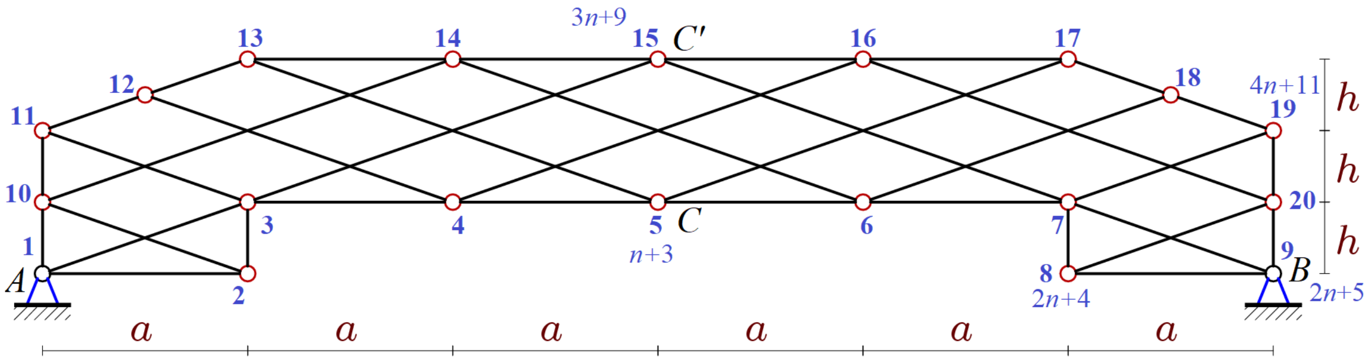

Fig. 2 – Numbering of truss nodes, $n = 2$

The coordinates of the nodes are as follows:

$$x_1 = y_1 = 0, x_2 = a, y_2 = 0,$$
$$x_{i+2} = x_{i+2n+8} = ai,\ y_{i+2} = h,\ y_{i+2n+8} = 3h,\ i = 1,..,2n+1,$$
$$x_{2n+4} = L_0 - a,\ y_{2n+4} = 0,\ x_{2n+5} = L_0,\ y_{2n+5} = 0,$$
$$x_{2n+6} = 0,\ y_{2n+6} = h,\ x_{2n+7} = 0,\ y_{2n+7} = 2h,$$
$$x_{2n+8} = a/2,\ y_{2n+8} = 5h/2,\ x_{4n+10} = L_0 - a/2,\ y_{4n+10} = 5h/2,$$
$$x_{4n+11} = x_{4n+12} = L_0,\ y_{4n+11} = 2h,\ y_{4n+12} = h.$$

The structure of the connection of individual rods into a lattice is specified by special vectors $\bar{q}_i$, $i = 1,...,\eta$, containing the numbers of the ends of the corresponding rods by analogy with the assignment of a list of edges of a graph in discrete mathematics. The following vectors correspond to the upper and lower chords of the truss:

$$\bar{q}_i = [i, i+1],\quad i = 1,...,2n+4,$$
$$\bar{q}_{i+2n+5} = [i+2n+5, i+2n+6],\ i = 1,...,2n+6.$$

The lattice (posts and braces) are set as follows:

$$\bar{q}_{i+4n+12} = [i+1, i+2n+5],\quad \bar{q}_{i+6n+18} = [i+2, i+2n+10],\ i = 1,...,2n+2.$$

The rods that model the fixed hinge supports correspond to four vectors:

$$\bar{q}_{\eta-3} = [1, 4n+13],\quad \bar{q}_{\eta-2} = [1, 4n+14],\ \bar{q}_{\eta-1} = [2n+5, 4n+15],\ \bar{q}_{\eta} = [2n+5, 4n+16].$$

Based on these data, the matrix **G** of the direction cosines of the efforts is formed. The system of equations of the equilibrium of nodes in the matrix version has the form:

GS=B,

where **B** is the vector of node loads, and **S** is the vector of unknown forces and support reactions. The length of these vectors is determined by the number *K* of truss nodes. The odd elements B_{2i-1} of the load vector contain projections of nodal loads onto the horizontal *x*-axis, and the even elements contain projections of forces applied to the truss onto the vertical *y*-axis. The elements of the matrix **B** are calculated using data on the lattice structure and node coordinates:

$$l_{x,i} = (x_{q_{i,1}} - x_{q_{i,2}})/l_i,\ l_{y,i} = (y_{q_{i,1}} - y_{q_{i,2}})/l_i,\ i = 1,...,h,$$

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where $l_i = \sqrt{l_{x,i}^2 + l_{y,i}^2}$ is the length of the rod with number *i*. The rows of the matrix **B** include the coefficients of the equations of projections on the *x* and *y* axes for one node:

$$G_{2h_{i,1}-1,i} = l_{x,i} / l_i,\; G_{2h_{i,1},i} = l_{y,i} / l_i,$$
$$G_{2h_{i,2}-1,i} = -l_{x,i} / l_i, G_{2h_{i,2},i} = -l_{y,i} / l_i.$$

The inverse matrix method in symbolic form with the help of Maple system operators is used to find a solution to system (1). The first trial calculations showed that for an odd number of panels *n* the determinant of the matrix **G** becomes zero. This indicates the kinematic degeneration of the structure. This fact is confirmed by the picture of possible node velocities for *n* = 1 (Fig. 3). Only three nodes receive possible velocities, the remaining nodes are motionless. The relationship between the velocities values specifies the position of the instantaneous velocities center: $U / a = 2V / c$, where $c = \sqrt{a^2 + h^2}$. The algorithm for sequentially calculating the velocities of the truss nodes using the kinematic relationships of plane motion is used [30].

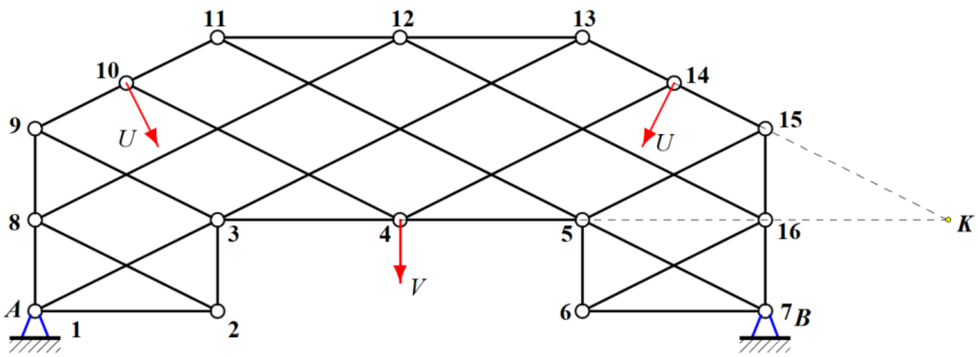

Fig. 3 – Possible velocities of truss nodes, $n=1$

2.2 The deflection

Two fixed hinged supports create external static indeterminacy of the structure. It is impossible to find the support reactions separately from the forces in the rods only from the equilibrium equations of the entire truss as a whole. The support reactions of a two-part structure, articulated by a hinge, on two fixed hinges are easily found by dividing the structure into two parts. Here, however, there is no articulating hinge. To determine the support reactions, it is necessary to compile the entire system of equilibrium equations of nodes, including the support nodes. For a truss loaded with a uniform nodal load along the lower chord, the nonzero components of the vector of the right side of the equilibrium equations of nodes have the form: $B_{2i} = -P,\; i = 2,..,2n+4$.

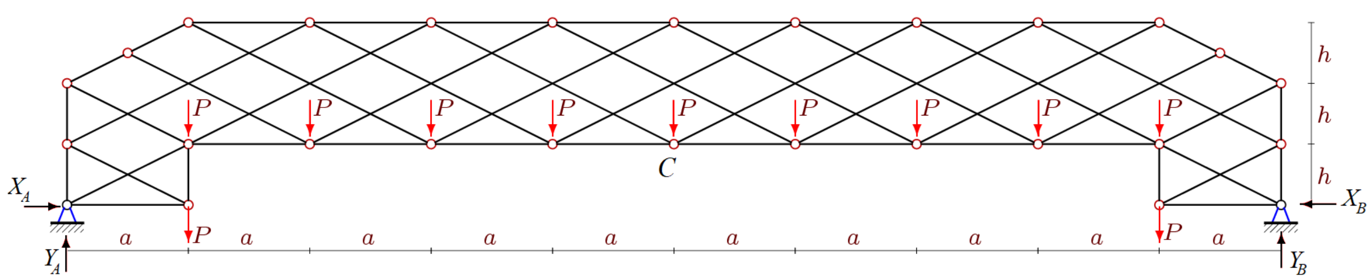

Fig. 4 – Load on the lower belt, $n=4$

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The vertical reactions of the supports calculated for the sequence of trusses are as follows: $Y_A = Y_B = 7P/2, 11P/2, 15P/2, 19P/2, \ldots$ The general formula for this value is obvious: $Y_A = Y_B = (4k+3)P/2$. The calculation shows that the horizontal reactions in this problem do not depend on the order of the truss: $X_A = X_B = 3aP/(2h)$. The deflection of the truss under the action of the load on the lower chord (Fig. 4) is estimated by the vertical displacement of the central hinge *C* using the Maxwell – Mohr's formula:

$$\Delta_k = \sum_{i=1}^{\eta} \frac{S_i s_i l_i}{EF} \ ,$$

where S_i is the force in the *i*-th rod from the external load, s_i — the force in the same rod from the action of a single vertical force on node *C, EF* is the rigidity of the rods, l_i is their length. Calculation of the deflection for trusses of different orders gives the following sequence:

$$\begin{aligned}
\Delta_1 &= P(9a^3 + 3c^3 + 23h^3)/(4h^2EF),\\
\Delta_2 &= P(51a^3 + 3c^3 + 11h^3)/(4h^2EF),\\
\Delta_3 &= P(617a^3 + 17c^3 + 63h^3)/(4h^2EF),\\
&\ldots\\
\Delta_{18} &= 3P(118385a^3 + 97c^3 + 25h^3)/(4h^2EF),\ldots
\end{aligned}$$

In general:

$$\Delta_k = P\left(C_1a^3 + C_2c^3 + C_3h^3\right)/(EFh^2). \tag{1}$$

To find the general term of this sequence using Maple, a sequence of length 18 was required. The most difficult task was to find the coefficient C_1. To determine it, using the operator **rgf_findrecur**, a recurrent equation of the ninth order was obtained, which is satisfied by this coefficient

$$C_{1,k} = C_{1,k-1} + 4C_{1,k-2} - 4C_{1,k-3} - 6C_{1,k-4} + 6C_{1,k-5} + 4C_{1,k-6} - 4C_{1,k-7} - C_{1,k-8} + C_{1,k-9}.$$

The solution to this equation gives the desired coefficient:

$$C_1 = (20k^4 + 8(5-4(-1)^k)k^3 - 2(24(-1)^k + 1)k^2 + 2(37(-1)^k - 11)k + 3(-1)^k + 15)/24. \tag{2}$$

Other coefficients are obtained similarly:

$$\begin{aligned}
C_2 &= (2k^2 + (2-6(-1)^k)k + 5(-1)^k + 1)/8,\\
C_3 &= (4(-2(-1)^k + 3)k + 3)/4.
\end{aligned} \tag{3}$$

Solution (1) with coefficients (2) and (3) has a quadratic asymptote with respect to the number of panels. Using Maple methods for the relative value $\Delta' = EF\Delta_k/(L_0P_{sum})$, where $P_{sum} = 2P(n+1)$ is the total truss load, one can find the limit $\lim_{k\to\infty}\Delta'/k^2 = 5a^2/(96h^2)$.

2.3 Natural frequency of oscillations

To calculate the first natural frequency of oscillations of a truss, assuming that the masses at the nodes perform only vertical movements, the Dunkerley estimate is used in a simplified form:

$$\omega_D = 1/\sqrt{M\sum_{i=1}^{K}\delta_i} = \sqrt{2/(M\delta^{\max}K)},$$

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where $d^{\max}$ is the greatest value of deflection $d_i,\ i = 1,..,K$ from the action of a vertical unit force on node *i* over all nodes, *m* is the mass in the node. In this case, the truss has two nodes with the greatest possible deflection. One node is in the middle of the lower chord and has the number *n*+3, the other with the number 3*n*+9 is in the middle of the upper chord. The calculation is carried out for kinematically permissible numbers of panels: $n = 2k,\ \ k = 1,2,...$. To clarify the result, the half-sum of these values is taken $d^{\max} = (d_{n+3} + d_{3n+9})/\ 2.$ The analytical value of the deflection is calculated using the Maxwell-Mohr formula as a sum over all the rods of the structure, including the four supporting ones:

$$d_b = \sum_{a=1}^{h} \left(S_a^{(b)}\right)^2 l_a\ /\ (EF), \tag{4}$$

where $S_a^{(b)},\ b = n + 3,\ 3n + 9,$ are the values of the force in the rod with number under the action of a single load on the node with number b , l_a is the length of the corresponding rod. The stiffnesses *EF* of all the rods of the truss are assumed to be the same.

Calculation of the sums in (1) for a sequence of trusses with an increasing number of panels yields the following formulas:

$$\begin{aligned}
d_1^{\max} &= \left(8a^3 + 3c^3 + 6h^3\right)/\ (8h^2EF),\\
d_2^{\max} &= \left(40a^3 + 5c^3 + 6h^3\right)/\ (8h^2EF),\\
d_3^{\max} &= \left(112a^3 + 7c^3 + 6h^3\right)/\ (8h^2EF),\\
d_4^{\max} &= 3\left(80a^3 + 3c^3 + 2h^3\right)/\ (8h^2EF),\\
d_5^{\max} &= \left(440a^3 + 11c^3 + 6h^3\right)/\ (8h^2EF),...
\end{aligned}$$

For an arbitrary number of panels, this expression has the form:

$$d_n^{\max} = \left(C_1a^3 + C_2c^3 + C_3h^3\right)/\,(EFh^2).$$

The coefficients in this expression are obtained in the Maple system by induction:

$$C_1 = k(k+1)(2k+1)/\,6,\ C_2 = (2k+1)/\,8,\ \ C_3 = 3/\,4. \tag{5}$$

Hence, the calculation formula for the first natural frequency is:

$$w_D = \frac{h}{2}\sqrt{\frac{EF}{M(2k + 3)\left(C_1a^3 + C_2c^3 + C_3h^3\right)}}\ . \tag{6}$$

3 Results and Discussion

The approximate dependence of the first natural oscillation frequency on the number of truss panels found should be compared with the minimum oscillation frequency obtained numerically taking into account all degrees of freedom of the system. Trusses made of steel rods with an elastic modulus of $E = 2.1{\cdot}10^5$ MPa and with a rod cross-sectional area $F = 9\,sm^2$ are considered. All masses concentrated at the nodes are the same $M = 100\,kg$, the panel size is $a = 3m.$ In Figure 5, the dotted lines w_1 correspond to the

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numerical solution obtained as the minimum natural frequency of the entire spectrum of natural frequencies of the truss.

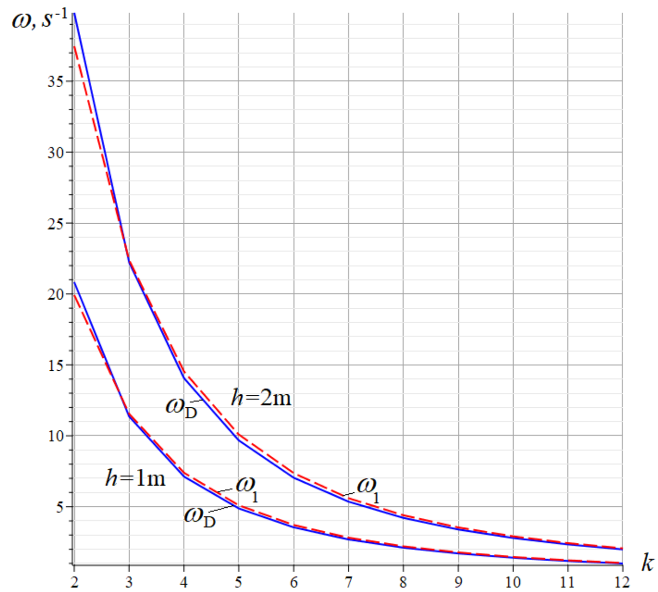

Fig. 5 – Dependence of the first frequency on the number of truss panels

The solution uses the Eigenvalues operator for calculating the eigenvalues of the matrix from the LinearAlgebra package of the Maple system. The curve $w*$ is plotted using the analytical solution (6) with coefficients (5). The numerical and analytical methods yield very close results; the curves, intersecting at *n*=1, almost merge. With an increase in the number of panels, the frequency of natural oscillations decreases monotonically. For a more accurate estimate of the error in the analytical solution, we can consider the dimensionless relative value $\varepsilon = |\omega_1 - \omega_D| / \omega_1$. The curves in Figure 5, plotted for two values of the truss height, show that starting with *k*=6, the error is within 5% and decreases with an increase in the number of panels.

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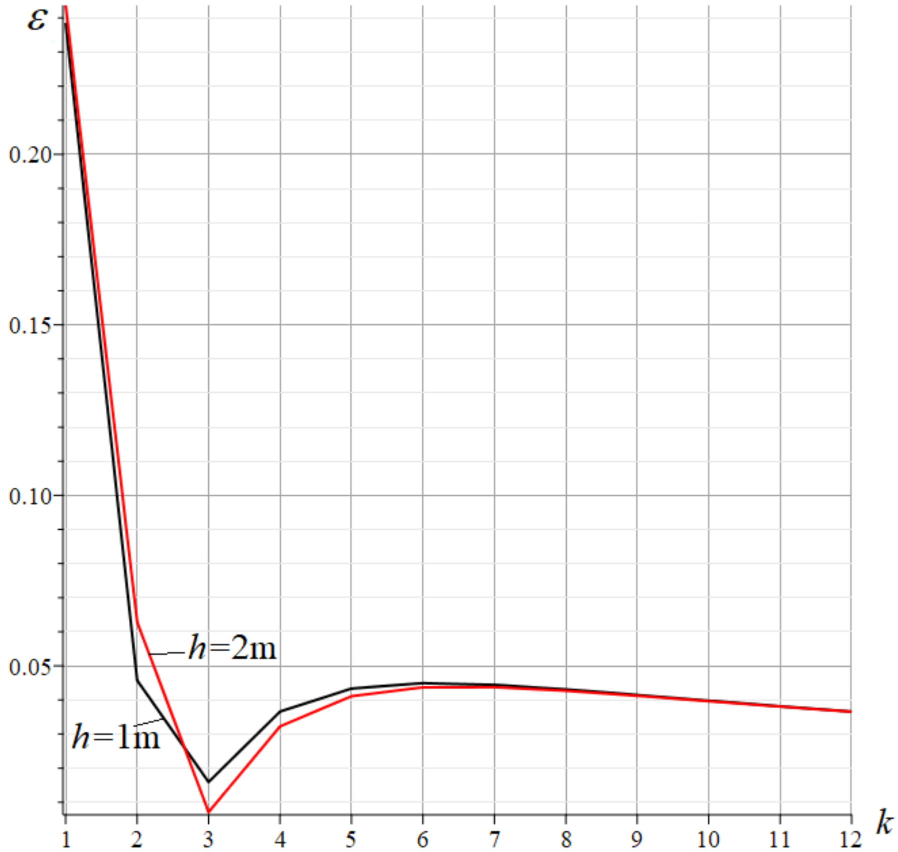

Fig. 5 – Relative error of method.

The height of the structure h has little effect on the accuracy of the analytical solution.

4 Conclusions

At the very beginning of the work on solving the problem, an unexpected feature of the proposed truss design was revealed. With an odd number of panels in half a span, the determinant of the matrix of the nodes equilibrium equations system turned out to be equal to zero for any dimensions of the truss. A thorough kinematic analysis of the scheme confirmed this by the presence of virtual velocities of nodes. Finding a pattern of velocity distribution turned out to be difficult. The above-mentioned author's kinematic algorithm for finding virtual velocities does not automatically provide a solution. It was necessary to try several options, assigning instantaneous angular velocities to some rods and leaving others motionless. Excluding odd numbers of panels from the calculations made it possible to solve the problem of oscillation frequency in analytical form. The basis for calculating the natural frequency was the simplified Dunkerley method, in which the sum of displacements is replaced by its average value. In this case, instead of one characteristic node, by the partial frequency of which the sought oscillation frequency of the entire structure is calculated, two nodes are used (in the middle of the upper and middle of the lower chord). The half-sum of the coefficients corresponding to these nodes provides a solution to the problem with high accuracy and in a compact form. The obtained solution turned out to be more accurate than the solution using the original Dunkerley method.

The following main conclusions can be drawn:

1. The estimate of the fundamental natural frequency obtained using the modified method is quite compact and at the same time provides good accuracy, increasing with the number of panels.
2. External static uncertainty did not prevent finding an analytical solution for support reactions and deflection values.

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3. Asymptotic characteristics of the solution to the deflection problem were found.

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